

# FORECASTING INFLATION WITH HIGH-FREQUENCY ASSET PRICE DATA

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## ABSTRACT

Mixed Data Sampling Regression Models (MIDAS) provide a means of incorporating the information that is included in high-frequency time series data when the variable that is to be explained is sampled at a lower frequency. This paper examines the properties of the model with the aid of a small Monte Carlo study and considers an extension to the model for the inclusion of multiple explanatory variables.

The paper then investigates the macroeconomic application of these tightly parameterised models in a country that conducts monetary policy with an inflation targeting framework. This empirical analysis involves the use of high-frequency forward-looking asset price data to forecast inflation.

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## 1. INTRODUCTION

Several econometric models have been constructed to make use of explanatory variables to explain, and in many instances forecast, economic time series. These regressions usually require that all variables are of the same frequency, which results in the aggregation of high frequency data and the loss of potentially useful information that could be used to identify the relationship between certain variables.

Mixed Data Sampling (MIDAS) models have been constructed to include explanatory variables that are sampled at a higher frequency than the variable that is to be explained. This implies that these models are able to include opulent information sets into the regression process by circumventing the *a priori* restriction that the dataset is of the same frequency.

The inclusion of high frequency explanatory variables into a MIDAS regression is made possible with the aid of a polynomial weighting function that is purely data driven. These polynomial weighting functions may assume various nonlinear shapes with the aid of only two parameters, and as a result, the model is not susceptible to unnecessary parameter proliferation.

The application of MIDAS models has been successfully applied to investigate the relationship between financial variables. This paper seeks to augment the application of the model into the economic sphere by using daily asset prices to forecast the monthly rate of inflation. To do so it has been necessary to extend the present framework of the model to incorporate multiple regressors.

In a review of the use of asset prices to forecast inflation, Stock and Watson (2003) suggest that such variables have little forecasting power. However, despite the poor results of previous studies, asset prices would nevertheless constitute a class of potentially useful predictors of the rate of inflation, since they have an inherent forward-looking perspective and are typically observed in real time with negligible measurement error.

The group of asset price regressors that have been used in previous studies include various interest rates, term spreads, default spreads, stock prices, dividend yields, exchange rates and various other variables. These regressors are usually sampled at a relatively low frequency (mostly quarterly), and hence, it would appear to be sensible to investigate whether the use of high frequency data would improve upon the results of such studies.

The outline of this paper is as follows: In the following section the basic characteristics of MIDAS models are described. In Section 3 the properties of the model are examined with the aid of a small Monte Carlo simulation. Section 4 includes a discussion on the inflation forecasting framework and describes the data that has been used in subsequent regressions.

In Section 5, the methodology that has been employed in the empirical analysis is described, whilst Section 6 includes details of the forecasts that were generated with the MIDAS technique. Section 7 concludes.

## 2. THE MIDAS MODEL

The application of simple MIDAS models was recently introduced as a tightly parameterised reduced form regression by Ghysels, Santa-Clara, Sinko and Valkanov (see, Ghysels, Santa-Clara, Sinko and Valkanov (2004), Ghysels, Santa-Clara and Valkanov (2004a), (2004b), (2004c)).

These MIDAS models share common features with distributed lag (DL) models and encounter a similar loss of efficiency when a lagged dependent variable is included as an explanatory variable (Ghysels, *et al.* (2004b)). However, in addition to sharing certain features with DL models, MIDAS models also have certain unique features that largely surround the specification of the polynomial weighting function.

To consider the unique characteristics of the MIDAS model, assume the dependent variable,  $y_t$ , is sampled at a fixed sampling frequency, which is termed the interval of reference. The explanatory variable is then denoted,  $x_t^{(m)}$ , where the value of  $m$  represents the frequency that the explanatory variable is observed during the interval of reference. Therefore, if the dependent variable is sampled monthly, and  $m = 21$ , then  $x_t^{(21)}$  represents data that is sampled daily (if there are 21 working days in the month).

With the aid of this notation, the simple linear MIDAS regression may be derived as:

$$y_t = \beta_0 + \beta_1 \left( B(1; \theta) x_{t-1/m}^{(m)} + B(2; \theta) x_{t-2/m}^{(m)} + \dots + B(K; \theta) x_{t-K/m}^{(m)} \right) + \varepsilon_t^{(m)}$$

where the parameter  $\beta_1$  captures the overall impact of lagged  $x_t^{(m)}$  on  $y_t$ , since the dependent variable is only sampled once between period  $t$  and period  $t+1$ .

This model may then be rewritten with the aid of the standard lag operator, so that:

$$y_t = \beta_0 + \beta_1 B(L^{1/m}; \theta) x_t^{(m)} + \varepsilon_t^{(m)}$$

where  $B(L^{1/m}, \theta) = \sum_{k=0}^K B(k; \theta) L^{k/m}$  must sum to unity and  $L^{1/m}$  represents the lag operator. This would imply that  $L^{1/m} x_t^{(m)} = x_{t-1/m}^{(m)}$ , and it would suggest that the lag coefficients in  $B(k; \theta)$  that relate to the corresponding lag operator  $L^{k/m}$ , may be parameterised as a function of the small *theta* vector.

With the aid of this introductory explanation, the parameterisation of the polynomial is considered before the method of parameter estimation is discussed. Thereafter, it is shown how this simple framework may be extended to include multiple regressors, before the main features of the model are summarized.

### 2.1. PARAMETERISATION OF THE POLYNOMIAL

One of the most important ingredients in a MIDAS regression is the parameterisation of the weighting function,  $B(k; \theta)$ , since a suitable parameterisation of the polynomial circumvents the problem of parameter proliferation and the difficulties involved in estimating the truncation point  $k_{max}$ . In addition, it is also important to ensure that the weighting functions always sum to unity, to facilitate the identification of the  $\beta_1$  coefficient.

Although there are many possible ways of parameterising  $B(k; \theta)$ , the following discussion is restricted to the methods that have been suggested in Ghysels, *et al.* (2004a) and Ghysels, *et al.* (2004). When using these polynomial functions, it is important to note that the most recently observed explanatory variable is expressed as element  $L^{1/m}$ , since it is expected that it would have the most influence over the variable that is to be explained. Similarly, the observation  $L^{21/m}$  would correspond to the first observation of the month.

The first of the parameterisation methods is termed the Exponential Almon lag, since it is closely related to the smooth polynomial Almon lag functions that are used to reduce

multicollinearity in the DL literature (see Almon (1965) and Judge, Griffiths, Carter Hill, Lutkepohl and Lee (1985)). It is often expressed as;

$$B(k; \theta) = \frac{e^{\theta_1 k + \dots + \theta_Q k^Q}}{\sum_{k=1}^K e^{\theta_1 k + \dots + \theta_Q k^Q}}$$

To understand the behaviour of this smooth polynomial lag function, consider the shape of the graphs that are included in figure 1, for different values of *theta* parameters. In the Exponential Almon lag models where there are only two *thetas*, various shapes can be created for the weighting functions. These include decreasing, increasing or single hump shaped patterns. If more humps are needed, it would be necessary to include more *thetas* in the Exponential Almon lag specification.

This example clearly shows how the MIDAS framework is able to include a nonlinear weighting function that does not encounter the proliferation of parameters that is often incurred when dealing with high frequency data. For instance, to capture daily fluctuations in a time series that extends over the last six months with the aid of a traditional DL model, it would be necessary to estimate a total of 126 parameters (6 x 21)  $b_k$  parameters.

After experimenting with various values of *theta* in the lag functions, it has been noted that the value of  $\theta_1$  exerts greater influence over the shape of the weight function than  $\theta_2$ , particularly when the values for the parameters are almost equal, or when both *thetas* are significantly greater than zero.

The second method of parameterisation is termed the Beta lag, as it is based on the beta function, which is used in Bayesian Econometrics to impose flexible, yet parsimonious prior distributions. It may be represented as;

$$B(k; \theta) = \frac{f(k/K; \theta_1, \theta_2)}{\sum_{k=1}^K f(k/K; \theta_1, \theta_2)}$$

$$\text{where } f(x; a, b) = \frac{x^{a-1} (1-x)^{b-1} \Gamma(a+b)}{\Gamma(a)\Gamma(b)}$$

$$\text{and } \Gamma(a) = \int_0^{\infty} e^{-x} x^{a-1} dx$$

To demonstrate the flexibility of the Beta lag polynomial consider the various shaped weight functions that are contained in figure 2, which have been generated with only two *theta* parameters. These graphs make it evident that these polynomial specifications are also able to capture the rich dynamic characteristics of the high frequency data in a manner that is flexible, simple, and parsimonious.

When using the Beta lag polynomial it is important to note that whilst it is highly flexible, it is limited to the inclusion of only two *theta* parameters. This means that the Exponential Almon lag specification is theoretically superior, since it only depends on the inclusion of  $Q$  parameters and is thus able to take on shapes that have multiple humps (Ghysels, *et al.* (2004)). However, since the objective of this study is to reduce the parameter proliferation, subsequent regressions have only included two *theta* parameters for each *beta* coefficient when using either Exponential Almon or Beta lag polynomials.

After selecting the appropriate functional form of  $B(L^{1/m}; \theta)$ , the lag length selection in the MIDAS model is purely data-driven. For instance, if explanatory power of the regressors was greater for realizations that were encountered towards the end of the month, then the weighting function of the Exponential Almon lag polynomial would look similar to that of the first graph in figure 1. Similarly, if the realizations of the regressors that were

generated during the middle of the month have more influence in explaining the regressand, then the Beta lag polynomial would look similar to the middle graph in figure 2.

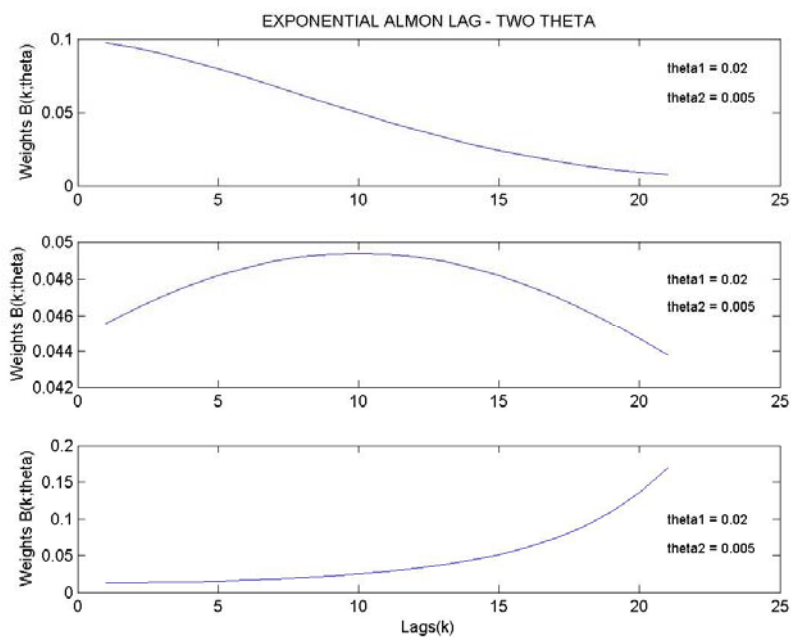


Figure 1: Exponential Almon lag model

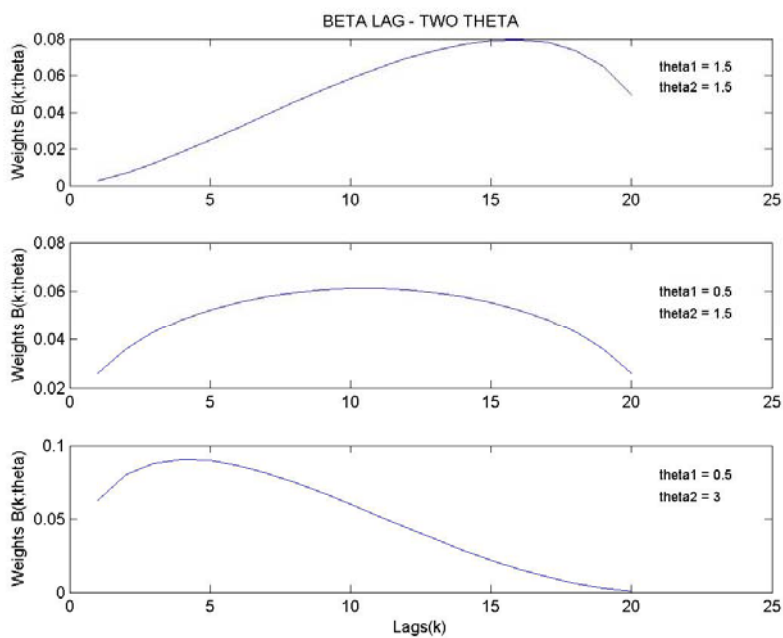


Figure 2: Beta lag model

## 2.2. METHOD OF PARAMETER ESTIMATION

Since the polynomial weighting function exhibits nonlinear characteristics, the method of nonlinear least squares (NLS) has been adopted to estimate the parameters in the MIDAS model. Although this technique is computationally intensive, the application of the Gauss-Newton and Levenburg-Marquardt algorithms has significantly reduced the amount of iterations that are needed to attain convergence.

In addition, Ghysels, *et al.* (2004a) have shown that the parameters that are estimated with this technique in a MIDAS framework are consistent with the parameters that are estimated with either the Maximum Likelihood (ML) or the General Method of Moments (GMM) procedures.

The method of NLS minimises the squared residuals with the aid of iterative search technique that is discussed in Wei (1990), Greene (2003), Hayashi (2000) and Judge, *et al.* (1985). To grasp the principles that espouse this procedure, consider the general representation of the regression model;

$$y_t = f(x_t, \beta) + e_t \quad t = 1, 2, \dots, n$$

where  $f$  is a general function of the explanatory variables  $x_t$  and the parameters  $\beta$ . If the model is linear in its parameters then the derivatives of  $f$  do not depend upon  $\beta$ . This allows for the general linear regression model to be expressed as;

$$\begin{aligned} y_t &= E(y_t | \mathbf{x}_t) + e_t \\ &= \beta_1 x_{t1} + \beta_2 x_{t2} + \dots + \beta_p x_{tp} + e_t \end{aligned}$$

where  $t = 1, 2, \dots, n$  and the corresponding matrix  $\mathbf{x}_t$  represents the independent explanatory variables  $x_{ti}$ . Following the standard procedures for a linear regression analysis, the estimated parameters and residual sum of squares are calculated as;

$$\begin{aligned} \hat{\beta} &= (\mathbf{x}'\mathbf{x})^{-1} \mathbf{x}'\mathbf{y} \\ S(\hat{\beta}) &= \sum_{t=1}^n (y_t - \hat{\beta}_1 x_{t1} - \hat{\beta}_2 x_{t2} - \dots - \hat{\beta}_p x_{tp})^2 \end{aligned}$$

Least squares estimation involves the selection of the parameter values that minimize the sum of square residuals. These parameter estimates could also be derived from an iterative search technique that calculates the residual sum of square as;

$$S(\hat{\beta}) = \sum_{t=1}^n \left[ y_t - \tilde{\beta}_1 x_{t1} - \tilde{\beta}_2 x_{t2} - \dots - \tilde{\beta}_p x_{tp} - (\hat{\beta}_1 - \tilde{\beta}_1) x_{t1} - (\hat{\beta}_2 - \tilde{\beta}_2) x_{t2} - \dots - (\hat{\beta}_p - \tilde{\beta}_p) x_{tp} \right]^2$$

where the initial guess for the value of the coefficients in matrix  $\beta$  is expressed as  $\tilde{\beta}$ .

Now consider a general expression that may be used to represent a MIDAS model that is nonlinear in its parameters;

$$y_t = f(\mathbf{x}_t, \boldsymbol{\psi}) + e_t \quad t = 1, 2, \dots, n$$

where the *psi* matrix,  $\boldsymbol{\psi} = (\psi_1, \psi_2, \dots, \psi_p)'$ , represents the parameters in the MIDAS regression, such that  $\boldsymbol{\psi} = (\beta; \theta)$ . In this instance, we assume that the derivatives of  $f$  are functions of  $\beta$ , which suggests that the model has nonlinear parameters, for which a closed form solution does not exist. This means that an iterative search technique would need to be employed to find the minimum residual sum of squares.

Given a vector that contains an initial guess of the  $\psi$  coefficients, the residuals may be calculated as  $\tilde{\mathbf{e}} = (\mathbf{y} - \tilde{\mathbf{y}})$ . This implies that the residual sum of squares may be determined as;

$$S(\tilde{\psi}) = \tilde{\mathbf{e}}' \tilde{\mathbf{e}} = (\mathbf{y} - \tilde{\mathbf{y}})' (\mathbf{y} - \tilde{\mathbf{y}})$$

where  $\tilde{\mathbf{y}} = f(\tilde{\psi})$  is a vector of predicted values that is obtained by replacing the unknown parameters with the initial guess values. To approximate the model  $f(\mathbf{x}_t, \psi)$ , the first order Taylor expansion for the initial value of  $\tilde{\psi}$  is used to find;

$$f(\psi) = f(\tilde{\psi}) + \mathbf{x}_{\tilde{\psi}} \delta$$

where  $\delta = (\psi - \tilde{\psi})$  and  $\mathbf{x}_{\tilde{\psi}} = \{x_{ij}\}$  is a  $n \times p$  matrix of the partial derivatives at  $\tilde{\psi}$  in the above linear approximation. To determine the value of  $\delta$  it is then possible to calculate;

$$\delta = (\mathbf{x}_{\tilde{\psi}}' \mathbf{x}_{\tilde{\psi}})^{-1} \mathbf{x}_{\tilde{\psi}}' \tilde{\mathbf{e}} = (\delta_1, \delta_2, \dots, \delta_{1p})$$

In a nonlinear model, the values in the matrix  $\mathbf{x}_{\tilde{\psi}}$  change for each iteration and the updated least squares estimates ( $\hat{\psi} = \tilde{\psi} - \delta$ ) are calculated per iteration. The estimates that are obtained from the first iteration are then used to replace the values that were used as the initial guess. Subsequent iterations continue until the difference between successive parameter vectors ( $\delta$ ) is small enough to assume that the *a priori* convergence criteria has been satisfied.

Hence, this procedure determines the least square estimates that minimize the residual sum of squares, where;

$$\tilde{\psi} = \underset{\psi}{\operatorname{argmin}} \sum_{t=1}^n (y_t - f(\mathbf{x}_t, \psi))^2$$

Although suitable interval estimates and confidence regions are available for the NLS estimators, Judge, *et al.* (1985) have noted that they are only applicable to estimates that satisfy conditions of consistency and asymptotic normality. This is an important finding, since econometricians are seldom presented with datasets that are large enough to satisfy this asymptotic criterion.

In addition, these authors have also suggested that current Monte Carlo studies that use this method of parameterisation lack generality when performed on small samples. With this in mind, a small problem specific Monte Carlo study has been performed to investigate the NLS method of parameterisation for the relevant sample size.

### 2.3. EXTENDING THE MODEL FOR MULTIPLE EXPLANATORY VARIABLES

When trying to forecast a variable such as inflation, it is often necessary to include several lags of an explanatory variable. To extend the MIDAS framework to include two lags of an explanatory variable it would be possible to derive the following specification;

$$y_t = \beta_0 + \beta_1 B(L^{1/m}; \theta_1) x_{t-1}^{(m)} + \beta_2 B(L^{1/m}; \theta_2) x_{t-2}^{(m)} + \varepsilon_t^{(m)}$$

In this instance, the regression includes explanatory variables that are sampled in period,  $t-1$  and  $t-2$ . Each lagged explanatory variable is assigned an independent *beta* coefficient,  $\beta_1$  and  $\beta_2$ . The respective vectors for the *theta* parameters,  $\theta_1$  and  $\theta_2$  are also independent to allow for the possibility that the polynomial has different weighting functions. Furthermore, it is worth noting that the values for the respective  $K$ 's in  $\sum_{k=1}^K B(k; \theta) L^{k/m}$  do not have to be equal for  $x_{t-1}$  and  $x_{t-2}$ , which would mean that it would be possible to combine explanatory variables that are sampled at different frequencies.

This would assume that the influence of the different lags of the explanatory variables is quite different. Of course, it would be possible to estimate only one *beta* parameter and one *theta* vector by merely enlarging the period that relates to an interval of reference, thereby combining the effect of  $x_{t-1}$  and  $x_{t-2}$  on a single set of parameters. This would mean that the value  $K$  would also need to increase as well.

To allow for the inclusion of several different explanatory variables into the MIDAS framework it would be necessary to extend the methodology that has been used above to derive the specification of a model that takes the following form;

$$y_t = \beta_0 + \beta_1 B(L^{1/m}; \theta_1) x_{1,t-1}^{(m)} + \beta_2 B(L^{1/m}; \theta_2) x_{2,t-1}^{(m)} + \varepsilon_t^{(m)}$$

In this case, it is assumed that  $x_1$  and  $x_2$  are two different explanatory variables. Once again, the values of the *theta* parameters in  $\theta_1$  and  $\theta_2$ , are assumed to take on independent values and are thus represented by two independent vectors for the parameters, which may take on different lag lengths.

Obviously, the above specifications may be extended to allow for the inclusion of more than two explanatory variables (or more than two lags), and these extensions would still incorporate the main features of the MIDAS model that may be summarized as:

1. The variables on the left-hand side and the variables on the right-hand side may be sampled at different frequencies.
2. The polynomial lag parameters are parameterized to be a function of *theta*. This allows the researcher to include additional information in the regression without including a proliferation of parameters (in the case where the  $q$  shape parameters are less than  $k$ ).
3. The model does not rely on an autoregressive scheme, where  $X_{t-k;t-k-1}^{(m)}$  is related to the lags of the left hand side variable.

### 3. MONTECARLO SIMULATIONS

To investigate the properties of the MIDAS model for the sample size that is used in the subsequent analysis, a small Monte Carlo analysis has been constructed in the manner described in Spanos (2000), Ross (1990) and Davidson and MacKinnon (1993).

This construction involves the simulation of the data generating process (DGP), which is assumed to represent a realization of the process that may be expressed as:

$$y_t = \alpha_y + \beta_y + B(L^{1/m}; \theta_1, \theta_2) x_t^m + \varepsilon_{y,t}$$

In this study, it is assumed that all the parameters in the simulation obey the normality assumptions, where  $\varepsilon_{y,t} \sim N(0; \sigma_y^2)$ . Therefore, the simulation of the DGP involves the following three steps:

1. Generate  $x_t$  according to an autoregressive process, for instance:

$$x_t = \alpha_x + \beta_x x_{t-1} + \varepsilon_{x,t} \quad \text{where } \varepsilon_{x,t} = N(0; \sigma_x^2).$$

2. Transform the respective  $x$  series into a matrix of explanatory variables,  $\mathbf{x}$ .
3. Generate values for the dependent variable according to the above equation for  $y_t$ .

When constructing a Monte Carlo study, due consideration should be given to the specification of the variables that assume different values, since their specification will influence the size of the study and the subsequent reporting requirements.

For example, if there are ten specifications for each of the six parameters ( $\theta$ 's,  $\alpha_y$ ,  $\beta_y$ ,  $\alpha_x$  and  $\beta_x$ ), the system would need to generate over 60 million simulations. To reduce the extent of this dimensional problem, the values for certain coefficients are calibrated to facilitate an investigation into the behaviour of the *thetas*.

This involves setting the remaining coefficients to fixed values that approximated those that were obtained from preliminary MIDAS regressions, where  $\alpha_y = 1$ ,  $\beta_y = 0.8$ ,  $\alpha_x = 1$  and  $\beta_x = 0.8$ . In addition, the variance of the error terms has also been set at a value equal to 0.5. The sample size was then set to include 238 observations in  $y$ , and 21 lags in  $x$ , since it is assumed that there are twenty-one working days in a month.

#### 3.1. VARYING THETAS IN THE EXPONENTIAL ALMON LAG

The first Monte Carlo simulation investigated the impact of varying the *theta* coefficients. To do so, the values for  $\theta_1$  were set to vary from -0.02 to 0.02, with a step size of 0.01 (i.e. 5 states). The values of the  $\theta_2$  coefficients were also set to vary from between -0.01 and 0, with a step size of 0.0005 (i.e. 21 states). To ensure that the study was not particularly sensitive to the inclusion of outliers, each state was generated on five successive occasions to obtain a mean estimate. This meant that this study involved a total of 525 simulations to obtain the bias,  $(\theta - \hat{\theta})$ , which is shown in Figure 3.

The simulation shows that the bias is relatively large, and at some points, it is even greater than the value of the coefficients. In addition, it was also noted that the  $t$ -statistics of these estimates are never above 2, which would suggest that the sample size is not capable of generating estimates that satisfy asymptotic principles.

A second simulation was then performed, where the value of  $\theta_2$  was set to vary from between -0.02 and 0.02, with a step size of 0.01, and the values of the  $\theta_1$  coefficient was set to vary from between -0.01 and 0, with a step size of 0.0005. In the second regression, the values are pretty similar to those of the first, as may be seen in Figure 4.

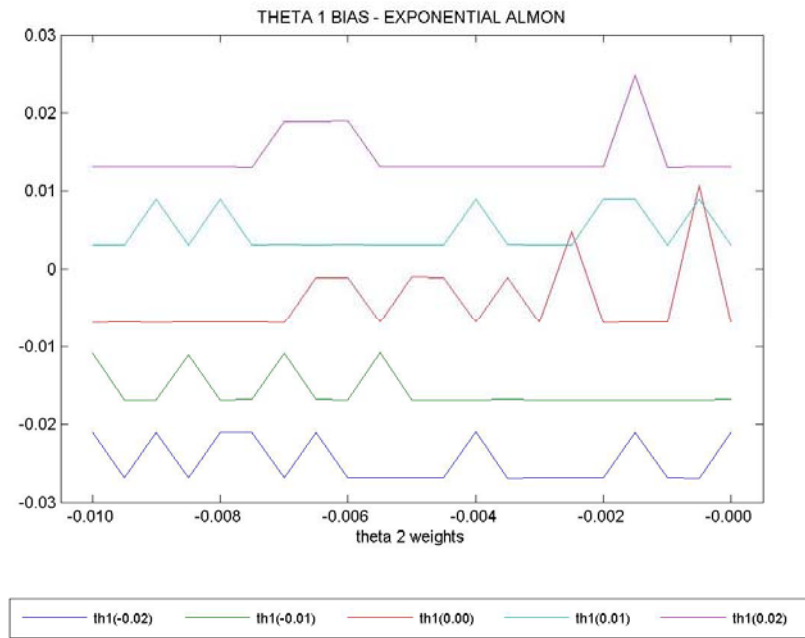


Figure 3: Bias in Theta 1 - Exponential Lag Model I

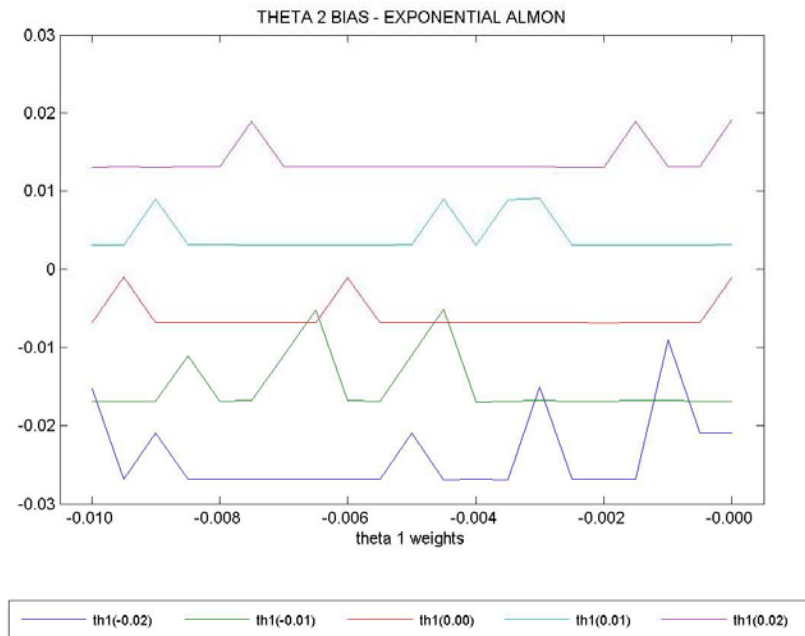


Figure 4: Bias in Theta 2 - Exponential Lag Model I

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### 3.2. VARYING THETAS IN THE BETA LAG

Following the disappointing results that were produced by the Exponential Almon lag, a similar study was conducted for the Beta lag. In this instance,  $\theta_1$  and  $\theta_2$  were both set to vary from between 1 and 1.5, partly due to the fact that the *theta* values in the Beta lag may not take on negative values. The step size for  $\theta_1$  was set at 0.25 (i.e. 5 states) and the step size for  $\theta_2$  was set at 0.02 (i.e. 21 states). This resulted in the generation of a further 525 simulations, which were performed to generate the results that have been shown in Figure 5.

This graph seems to suggest that a bias in the *theta* coefficients is negligible. To ensure that this is indeed the case, Figure 6 graphs the same data on separate axes and it is noted that although a bias is still present, it is very small (note the scale of the vertical axes).

In addition, it was also worth noting that the values of the Beta lag test statistics improved upon those that were produced by the Exponential Almon lag, as the standard errors are not nearly as large and the *t*-statistics of the *beta* coefficients were almost always significant at the 5% level.

However, one disappointing feature of the Beta lag models was that the positive starting values seemed to influence the results, which would suggest that the method of parameterisation is prone to finding local minima. In an attempt to minimize the impact of this finding, a very small random component was attached to the starting values of the coefficients that were used to produce these results. This would imply that the analysis has been performed with multiple starting values since the result of each simulation is calculated from the mean of five iterations.<sup>1</sup>

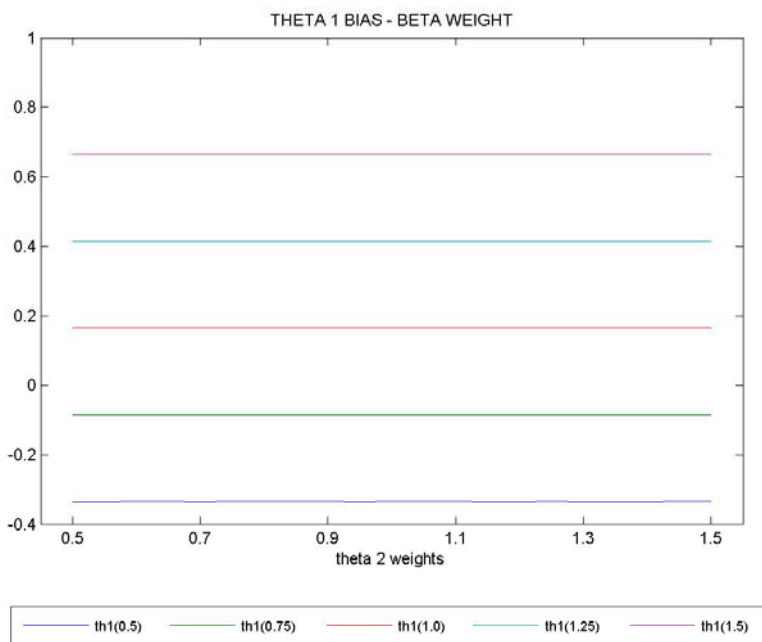


Figure 5: Bias in Theta 1 – Beta Lag Model I

<sup>1</sup> The full results from both these simulations have been included in the appendix.

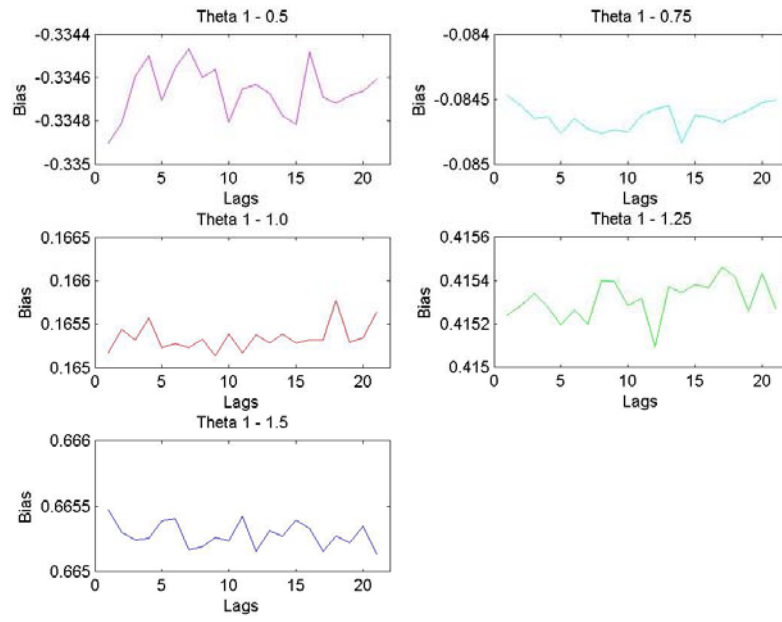


Figure 6: Bias in Theta 1 - Beta Lag Model I

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#### 4. INFLATION FORECASTING WITH MIDAS MODELS

The use of an inflation targeting framework for conducting monetary policy in both developed and developing countries has become increasingly popular since 1990<sup>2</sup>. In this forward-looking policy framework, monetary policy is largely guided by a conditional forecast of inflation, largely due to the fact that central banks have little control over the observed rate of inflation.

Svensson (1996) argues that the use of such an intermediate target is ideal, since it is transparent, comprehensible, easily observable and highly correlated with the ultimate goal. In addition, since the central bank is able to exert better control over the intermediate target (which is not susceptible to the influence of unexpected shocks) it would be poised to conduct reliable policy.

The use of an inflation forecast is not only of importance from the perspective that it is responsible for guiding monetary policy, but it also provides the basis for the monitoring of the policy process (du Plessis (2003)). This monitoring process has a forward looking *ex ante* perspective, which determines whether the stance of monetary policy is appropriate at the start of a period in time. This usually involves a comparison of the conditional forecast with the target rate.

In addition, the monitoring process also includes a backward looking *ex post* evaluation, to determine whether appropriate monetary policy was responsible for the success or failure of hitting the forecasted target. This form of public monitoring is essential for the transparency and accountability of monetary policy. If it is undermined, the central bank would incur a loss of credibility, which would hamper its ability to anchor inflation expectations (Bernanke and Woodford (1997)).

The relationship between asset prices and inflation is embodied in many of the foundations of macroeconomic theory (Stock and Watson (2003)). For instance, Fisher's theory suggests that the nominal interest rate reflects the real rate plus the expected rate of inflation.

In addition, Clews (2002) has argued that asset prices have a direct impact on monetary policy, since policy committees, such as the Monetary Policy Committee of the Bank of England are usually extensively briefed on asset market developments before they make policy decisions. This may also be evidenced by the fact that the quarterly Inflation Report of the Bank of England includes a detailed report on monetary and asset prices, which would suggest that changes in asset prices are taken into account when conducting monetary policy.

However, although economic theory would suggest that an obvious relationship exists between asset prices and the rate of inflation, empirical studies have not been able to confirm its existence. This may be due to three critical reasons that Svensson (1999) has identified. Firstly, it has been noted the rate of inflation subjected to long and variable lags in the monetary transmission mechanism. Secondly, it has been suggested significant uncertainty surrounds the respective channels of the monetary transmission mechanism. And finally, it is argued that there is also significant uncertainty that surrounds the identification of the present state of the economy and the occurrence of likely future shocks, which would have a significant impact on any forecast.

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<sup>2</sup> The developed nations include; Australia, Canada, Czech Republic, Hungary, Iceland, Israel, Korea, New Zealand, Norway, Poland, Sweden and the United Kingdom, whilst the developing nations include; Brazil, Chile, Columbia, Mexico, Thailand and South Africa

In the analysis that has been conducted below, the rate of inflation for the United Kingdom<sup>3</sup> has been forecast in a MIDAS model that accommodates explanatory variables that are sampled at a higher frequency than the variable that is to be explained. The group of explanatory variables include the rate of interest, the exchange rate, the yield differential between index-linked and conventional bonds, the stock price index, and the term spread.

Along with the dependent variable, the explanatory variables were transformed according to similar procedures that were used in papers such as Stock and Watson (2003), Goodhart and Hofmann (2000), and Rudebusch and Svensson (1999).

#### 4.1. DEPENDENT VARIABLE: THE RATE OF INFLATION

The rate of inflation was derived from monthly consumer price indices, for the period 1 January 1985 to 30 April 2005. The inflation forecasting target in the United Kingdom is currently based on the Consumer Prices Index (CPI)<sup>4</sup>. This index was constructed in response to the need for an internationally comparable measure that is based on consistent principles throughout Europe (as required by the Maastricht Treaty).

Since the index was only launched in 1997 (with estimates for the rate of inflation prior to this date extending to January 1988) it was necessary to supplement the series with data from other measures of inflation to obtain a sample that started on 1 January 1985. This involved deflating the CPI by the equivalent percentage change in the Retail Price Index, which was the main measure of consumer prices during the earlier period.

The data for the monthly CPI index was obtained from the Bank of England and the data for the Retail Price Index was obtained from EuroStat (the statistical office of the European Union). To derive a rate of inflation that is free of unit roots, the index was transformed by taking the log of first difference.

#### 4.2. EXPLANATORY VARIABLES

The data for each of the explanatory variables was obtained on a daily frequency. Each of the time series was then transposed to create a matrix, where each row represented a month and the columns represented the days in each of the corresponding months.

Therefore, element  $a_{11}$  would refer to the last working day in the first month of the first year, and element  $a_{23}$  would refer to the third last working day in the second month in the first year.

##### 4.2.1. NOMINAL SHORT-TERM INTEREST RATE

Changes to the short-term interest rates affect the rate of inflation through the monetary transmission mechanism, by influencing the level of demand in the economy relative to the economy's capacity to supply.

It has been suggested that lower interest rates encourage people to spend more and save less, thus stimulating economic activity (Taylor (1995)). In addition, raising the rate of interest would not only reduce disposable income as a result of the increase in mortgage repayments, but it would also affect house prices and the ability of households to raise further capital, since they would find it more difficult to gather sufficient collateral (Meltzer (1995)).

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<sup>3</sup> Current research activities include an investigation into the use of these models to forecast inflation in South Africa.

<sup>4</sup> Prior to December 1993, this index was referred to as the Harmonised Index of Consumer Prices (HICP)

Similar arguments have also been made regarding the adverse effects that high interest rates have on the quality of firms' balance sheets (Bernanke and Gertler (1995)), whilst the influence of interest rates on exchange rates has been well documented (exchange rate channel).

The main monetary policy instrument of the Bank of England is the repo rate, for which data is available from 6 May 1997. To extend this time series, the Bank of England supplemented the data with the Minimum Band 1 Dealing Rate, which reflects the minimum published rate that the bank discounted bills to relieve money market shortages. This has allowed for the construction of a time series that was sourced from the Bank of England, which may be used to describe nominal short-term interest rates for the period 1 January 1985 to 30 April 2005.

#### **4.2.2. THE EFFECTIVE EXCHANGE RATE**

The exchange rate has a significant impact on economic activity, since it affects the rate of return that is earned on assets denominated in sterling, relative to the rate of return that is earned on assets denominated in other currencies (Obstfeld and Rogoff (1995)). Similar arguments have also been made regarding the effect of the exchange rate on the competitiveness of UK output, which influences the level of economic activity. In addition, the exchange rate will also influence the price at which goods and services are imported into the country, which in turn directly affects the rate of domestic inflation.

An effective exchange rate measures the value of a currency against a trade-weighted basket of other currencies, relative to the base date. It is calculated as a weighted geometric average, expressed in the form of an index. This data was obtained from the Bank of England for the period 1 January 1985 to 30 April 2005 and its calculation was based on the principles of the International Monetary Fund's effective exchange rate index. The time series was transformed by taking the log of first difference.

#### **4.2.3. THE EXPECTED RATE OF INFLATION**

The expected rate of inflation is derived from the Fisher relationship, which suggests that the nominal rate of interest embodies the real interest rate plus a compensation for the erosion of the purchasing power by inflation (Fisher (1911)). It is calculated as the difference between the yield in the index-linked gilt market (which is used to derive the real interest rate) and the yield of a similar instrument in the conventional gilt market (which is used to determine the nominal interest rate).

The relative yields at which these two forms of debt trade would obviously depend (in part) on the expectations of future movements in the rate of inflation. The source of this data is the Bank of England, for both five and ten year instruments, for the period 1 January 1985 to 30 April 2005.

#### **4.2.4. FTSE 100 SHARE PRICE INDEX**

It has been argued that if the stock market price index reflects the expected discounted value of future earnings, then the rate of growth in the price index should be positively related to economic activity (Clews (2002)). When the prices of equities rise, individuals experience an increase in wealth, which may induce them to spend more, as the need to save would be less apparent (wealth effect). In addition, both businesses and individuals would be able to raise additional capital to finance further investment, as both collateral and the strength of their balance sheets would improve.

The time series was transformed by taking the logarithm of the first difference of the FTSE 100 price index for the sample period 1 January 1985 to 30 April 2005. The data was obtained from EconStats.

#### 4.2.5. TERM SPREAD

The term spread is used as an indicator of a reduction in inflationary pressure that may follow an economic recession (Frankel (1982)). Tight monetary policies may cause an inverted yield curve when short-term interest rates rise relative to long-term rates. Since relatively high short-term interest rates would normally hinder economic activity, we would expect to see a decline in economic output when the yield curve is inverted.

This inference could also be made by considering that long-term interest rates are frequently used to reflect the expected future short-term interest rate. Therefore, an inverted yield curve would suggest that current short-term interest rates are above the expected rate, and as a result, a subsequent decline in economic activity (and the accompanying rate of inflation) would be expected to ensue.

The term spread has been calculated as the difference between the yields of the ten year UK Government Treasury bond and the three month daily sterling certificate of deposit, for the period 1 January 1985 to 30 April 2005. The data was sourced from the Bank of England.

## 5. MIDAS ESTIMATION

Owing to the complexities of nonlinear empirical modelling, it has been suggested that the specific-to-general approach should be used in preference to the general-to-specific approach (see, Granger (1993) and Franses and van Dijk (2004)). This methodology has been adopted in this analysis, where each explanatory variable is used in a separate regression to explain the rate of inflation. Each explanatory variable is lagged from between one and six months, which implies that six regressions have been performed for each explanatory variable.

Following the estimation of each of these single explanatory variable models, the optimal monthly lag length for each variable is determined in accordance with the information criterion that was calculated for each model. These simple optimal models were then combined to estimate a multiple explanatory variable MIDAS regression.

These regressions were performed using both Exponential Almon and Beta lag polynomials.

### 5.1. SINGLE EXPLANATORY VARIABLE EXPONENTIAL ALMON LAG MODELS

In each of the single explanatory variable models, a separate *beta* has been included for each monthly lag and two *thetas* were included for each *beta*.

Although the  $r^2$  statistic has been calculate in accordance with the traditional methodology, the Akaike Information Criteria (AIC) and the Bayesian Information Criteria (BIC) have been calculated so that penalties are imposed for the inclusion of additional variables (or monthly lags). This is in contrast to the traditional method, which imposes penalties on including additional parameters.

This would imply that the information criterion is calculated in the following manner:

$$AIC = n^{-1} \log \hat{\sigma}^2 + \frac{2k}{n}$$

$$BIC = n^{-1} \log \hat{\sigma}^2 + \frac{k}{n} \log(n)$$

where  $n$  is the number of monthly observations (rows in  $\mathbf{x}$ ), and  $k$  is the number of daily lags (columns in  $\mathbf{x}$ ). Table 1 displays the summarised results of the thirty-six simple MIDAS regressions that make use of the Exponential Almon lag polynomial.

### 5.2. SINGLE EXPLANATORY VARIABLE BETA LAG MODELS

Similar regressions were performed for the MIDAS regressions that utilize a Beta lag polynomial function. These simple models were also used to identify the optimal monthly lag lengths for each variable that was included in a subsequent MIDAS regression for multiple explanatory variables. The results of these models are presented in Table 2.

	R <sup>2</sup>	AIC	BIC
Short-term interest rate:			
- 1 lag	0.000000	-2582.50	-2579.00
- 2 lags	0.100190	-2597.60	-2576.80
- 3 lags	0.108980	-2599.90	-2579.10
- 4 lags	0.110770	-2600.40	-2579.60
- 5 lags	0.106150	-2599.20	-2578.40
- 6 lags	0.110910	-2600.40	-2579.60
Effective exchange rate:			
- 1 lag	0.000000	-2572.50	-2551.60
- 2 lags	0.008374	-2574.50	-2553.60
- 3 lags	0.013718	-2575.80	-2554.90
- 4 lags	0.053877	-2585.70	-2564.80
- 5 lags	0.066440	-2588.80	-2568.00
- 6 lags	0.062213	-2587.80	-2566.90
Expected inflation (5 Years):			
- 1 lag	0.000000	-2572.50	-2551.60
- 2 lags	0.074062	-2590.80	-2570.00
- 3 lags	0.074058	-2590.80	-2570.00
- 4 lags	0.075626	-2591.20	-2570.40
- 5 lags	0.119310	-2602.70	-2581.90
- 6 lags	0.109350	-2600.00	-2579.20
Expected inflation (10 Years):			
- 1 lag	0.000000	-2572.50	-2551.60
- 2 lags	0.042412	-2582.80	-2562.00
- 3 lags	0.042744	-2582.90	-2562.00
- 4 lags	0.051420	-2585.00	-2564.20
- 5 lags	0.056405	-2586.30	-2565.50
- 6 lags	0.064480	-2588.30	-2567.50
FTSE 100 price index:			
- 1 lag	0.000000	-2572.50	-2551.60
- 2 lags	0.024307	-2578.30	-2557.50
- 3 lags	0.035456	-2581.10	-2560.20
- 4 lags	0.057549	-2586.60	-2565.80
- 5 lags	0.064677	-2588.40	-2567.60
- 6 lags	0.127590	-2605.00	-2584.10
Term Spread:			
- 1 lag	0.000000	-2572.50	-2551.60
- 2 lags	0.063359	-2588.10	-2567.20
- 3 lags	0.068060	-2589.30	-2568.40
- 4 lags	0.110960	-2600.50	-2579.60
- 5 lags	0.122410	-2603.60	-2582.70
- 6 lags	0.143460	-2609.30	-2588.50

Table 1 – Single explanatory variable Exponential Almon lag models

	R <sup>2</sup>	AIC	BIC
Short-term interest rate:			
- 1 lag	0.000000	-2582.50	-2579.00
- 2 lags	0.099890	-2597.50	-2576.70
- 3 lags	0.098170	-2597.10	-2576.20
- 4 lags	0.110620	-2600.40	-2579.50
- 5 lags	0.105640	-2599.00	-2578.20
- 6 lags	0.105360	-2599.00	-2578.10
Effective exchange rate:			
- 1 lag	0.000000	-2572.50	-2551.60
- 2 lags	0.002777	-2573.10	-2552.30
- 3 lags	0.004848	-2573.60	-2552.80
- 4 lags	0.007811	-2574.30	-2553.50
- 5 lags	0.010855	-2575.10	-2554.20
- 6 lags	0.015803	-2576.30	-2555.40
Expected inflation (5 Years):			
- 1 lag	0.000000	-2572.50	-2551.60
- 2 lags	0.070382	-2589.80	-2569.00
- 3 lags	0.068173	-2589.30	-2568.50
- 4 lags	0.068735	-2589.40	-2568.60
- 5 lags	0.085562	-2593.50	-2572.70
- 6 lags	0.071801	-2590.20	-2569.40
Expected inflation (10 Years):			
- 1 lag	0.000000	-2572.50	-2551.60
- 2 lags	0.042038	-2582.70	-2561.90
- 3 lags	0.039737	-2582.10	-2561.30
- 4 lags	0.041202	-2582.50	-2561.70
- 5 lags	0.058350	-2586.60	-2565.70
- 6 lags	0.055721	-2586.00	-2565.10
FTSE 100 price index:			
- 1 lag	0.000000	-2572.50	-2551.60
- 2 lags	0.015217	-2576.10	-2555.30
- 3 lags	0.008558	-2574.50	-2553.70
- 4 lags	0.032780	-2580.40	-2559.60
- 5 lags	0.024948	-2578.50	-2557.70
- 6 lags	0.056660	-2586.40	-2565.50
Term Spread:			
- 1 lag	0.000000	-2572.50	-2551.60
- 2 lags	0.057007	-2586.40	-2565.60
- 3 lags	0.059769	-2587.10	-2566.30
- 4 lags	0.057618	-2586.60	-2565.80
- 5 lags	0.073981	-2590.70	-2569.80
- 6 lags	0.052962	-2585.40	-2564.60

Table 2 – Single explanatory variable Beta lag models

### 5.3. MULTIPLE EXPLANATORY VARIABLES MODELS

#### 5.3.1. EXPONENTIAL ALMON LAG MODEL

The results from the single explanatory variable MIDAS regressions are used to formulate a multiple explanatory variable MIDAS model with the aid of the AIC, which determines the specification for the amount of monthly lags. In addition, the AIC is also used to determine whether five or ten year instruments should be used for the expected rate of inflation.

This resulted in the inclusion of the following variables:

- short-term nominal interest rate (6 lags),
- exchange rate (5 lags),
- expected rate of inflation – 5 year instruments (5 lags),
- FTSE 100 price index (6 lags),
- term spread (6 lags).

##### 5.3.1.1. RESULTS FROM EXPONENTIAL ALMON LAG MODEL

The strength of the relationship between the actual rate of inflation and the rate that was predicted with the aid of the five explanatory variables has been included in Figure 7. This graph suggests that the regression describes a weak relationship between left and right hand side variables.

For comparative purposes, a DL model was specified for the same explanatory variables. The resulting test statistics, which were calculated using heteroscedasticity and autocorrelation consistent (HAC) standard errors, in accordance with the method stipulated by Newey and West (1987), have been included in Table 3.

The results from both of these regressions seem to suggest that the MIDAS regressions have more explanatory power, as the  $r^2$  statistic is one and a half times that of the DL model. However, it is worth noting that these five explanatory variables are still only able to explain just over 36% of the rate of inflation in the MIDAS regression. In addition, the signs of the *beta* coefficients that are contained in Table 4 are also of some concern, since one would not expect that the relationship would be positive in one month and negative in the month thereafter.

Similar concerns also surround the estimation of the *theta* coefficients, which are represented by the weighting functions that are included as Figures 8 through 12. For example, the weighting functions for short-term interest rate suggest that whilst the majority of the weight in the first, fifth and sixth lag is placed on the observations that occur towards the beginning of the month, the majority of the weight in the second and fourth lag is placed on the observations that occur in the middle of the month.

A similar observation may also be made for the exchange rate, where the majority of the weight in the first and fifth lag has been placed on the observations in the early period of the month, whilst the weight is largely placed on the middle of the month during the second, third and fourth periods.

Therefore, although the results from the MIDAS Exponential Almon lag model are encouraging in terms of the greater explanatory power, the values of the beta coefficients and the graphs of the weighting functions seem to suggest that in seeking to accommodate a greater amount of information, the model has also possibly incorporated a significant amount of additional noise.

In addition, the inability of the model to derive accurate standard errors is concerning, particularly when the theta coefficients place the entire weight of the lag on a single

observation, since this would suggest that the relative merit of including the remaining observations for this variable (or lag) is no different than zero.

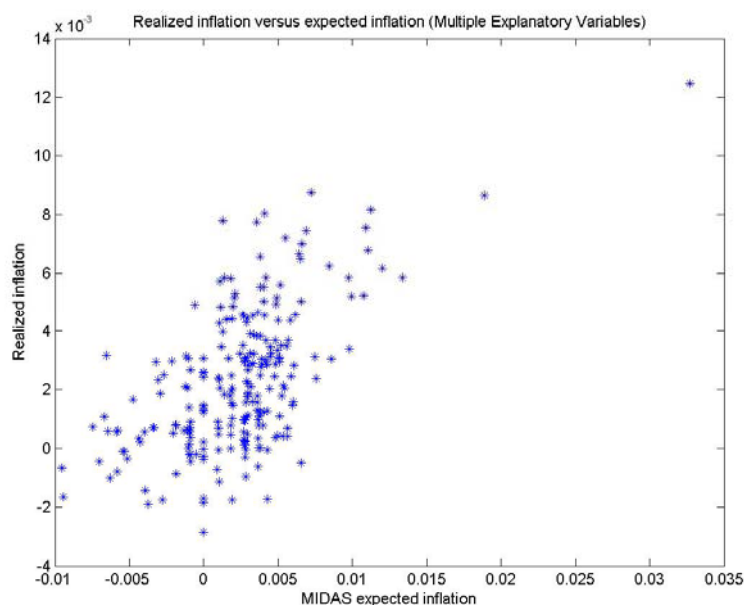


Figure 7 – Relationship in the MIDAS Exponential Almon lag model

	$R^2$
MIDAS Almon model	0.36238
Distributed lag model	0.22296

Table 3 – MIDAS Exponential Almon lag models and Distributed Lag models

	Short-term interest rate	Exchange Rate	Expected Inflation (5yr)	FTSE 100 Index	Term spread
<b>Beta coefficients</b>					
- 1 lag	-0.000111	0.014145	0.000549	0.008848	0.002952
- 2 lags	0.004020	0.202810	-0.001080	0.214500	-0.000132
- 3 lags	-0.002335	-0.162200	0.000980	-0.028391	-0.002602
- 4 lags	-0.005122	-0.263910	0.001681	0.084733	-0.007169
- 5 lags	0.002636	0.071530	-0.001564	-0.066276	0.005874
- 6 lags	0.000958			-0.272250	0.000910
AIC	-2538.40				
BIC	-2635.60				
$R^2$	0.36238				

Table 4 – MIDAS Exponential Almon lag beta coefficients

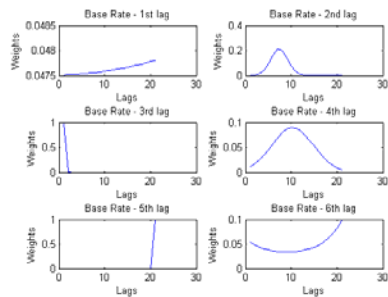


Figure 8 – Short-term interest rate weight

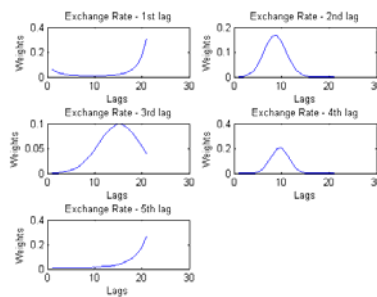


Figure 9 – Exchange rate weight

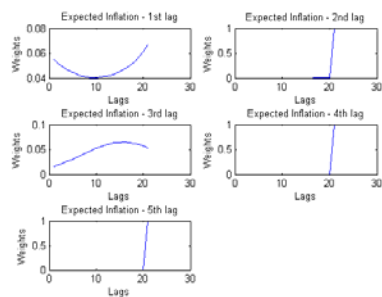


Figure 10 – Expected inflation (5yr) weight

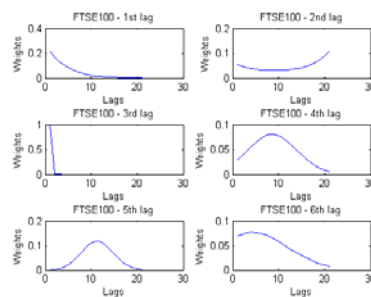


Figure 11 – FTSE 100 index weight

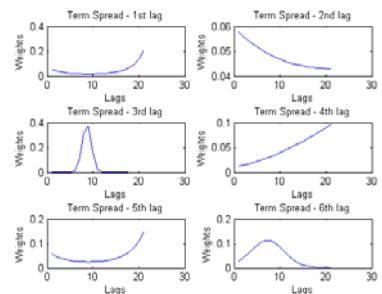


Figure 12 – Term spread weight

### 5.3.2. BETA LAG MODEL

In a manner that is similar to that which has been performed above, the AIC that was generated in the simple MIDAS beta lag models has been used to identify the optimal monthly lag lengths for the explanatory variables. This resulted in the inclusion of the following variables:

- short-term nominal interest rate (4 lags),
- exchange rate (6 lags),
- expected rate of inflation – 5 year instruments (5 lags),
- FTSE 100 price index (6 lags),
- term spread (5 lags).

### 5.3.2.1. RESULTS OF BETA LAG MODEL

The graph in Figure 13 seems to suggest that there is a relationship between the actual rate of inflation and the rate that has been predicted from the five explanatory variables, although the relationship does not seem to be as strong as in the case of the Exponential Almon model.

This is confirmed in Table 5, where it is noted that the explanatory power is slightly weaker than that of the DL model, which was estimated using the same lags in the explanatory variables. Once again, the signs of the *beta* coefficients that are contained in Table 4 are concerning, since they continue to fluctuate between positive and negative values in successive months. Furthermore, the very small values of the exchange rate coefficients are also somewhat disturbing.

However, unlike the Exponential Almon lag models, the *theta* coefficients in the Beta lag models appear to be more accurate. In almost all of the models, the weighting functions appear to exhibit a downward sloping pattern, which would suggest that the observations during the latter part of the month exert more influence over the relationship than the observations that occur during early part of the month. The graphs of the weighting functions are included as Figures 14 through 18.

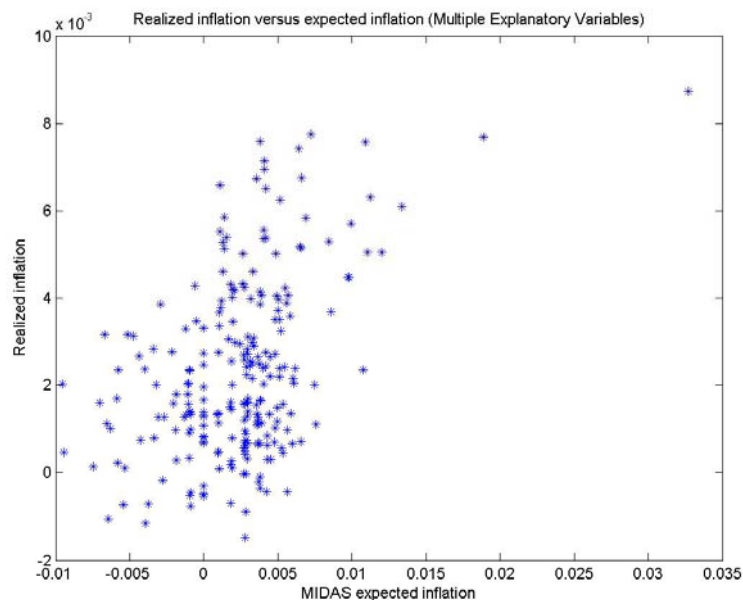


Figure 13 – Relationship in the MIDAS Beta lag model

	$R^2$
MIDAS Beta model	0.21129
Distributed lag model	0.22296

Table 5 –MIDAS Beta lag and Distributed lag models

	Short-term interest rate	Exchange Rate	Expected Inflation (5yr)	FTSE 100 Index	Term spread
<b>Beta coefficients</b>					
- 1 lag	-0.000120	0.000000	-0.026468	-0.244380	0.001771
- 2 lags	0.000800	0.000000	0.156360	0.000724	-0.000797
- 3 lags	0.000140	0.000000	0.092697	0.000762	-0.002029
- 4 lags	-0.000572	0.000000	0.145720	-0.000107	-0.000555
- 5 lags		0.000000	0.013074	-0.000833	0.001740
- 6 lags		0.000000		-0.000308	
AIC	-2589.00				
BIC	-2498.70				
$R^2$	0.21129				

Table 6 – MIDAS Beta lag beta coefficients

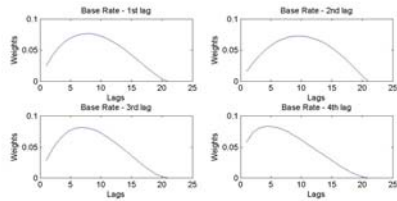


Figure 14 – Short-term interest rate weight

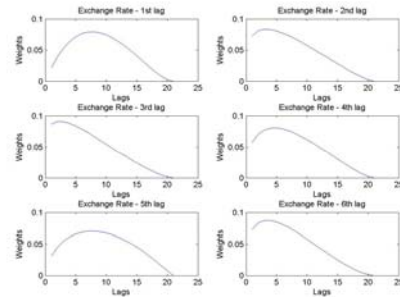


Figure 15 – Exchange rate weight

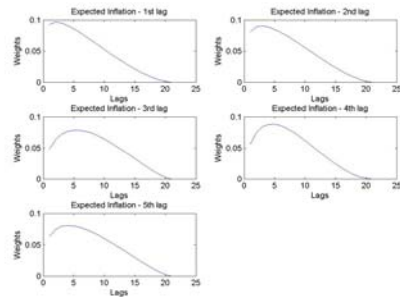


Figure 16 – Expected inflation (5yr) weight

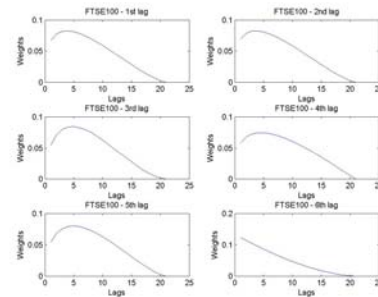


Figure 17 – FTSE 100 index weight

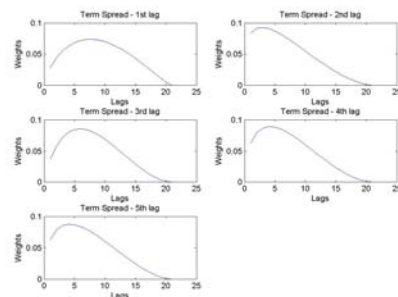


Figure 18 – Term spread weight

## 6. FORECASTING INFLATION

### 6.1. FORECASTING ONE MONTH AHEAD

In the final section of this paper, the multiple explanatory variable versions of the MIDAS models are used to forecast the rate of inflation in a traditional  $h$  step-ahead forecasting model. This entailed the estimation of the parameters for the out-of-sample period, which ends in December 2003. In addition to investigating the forecasting power of the model, this methodology may also be used to identify any overfitting or model instability in the regression.

The forecast values for the following periods were then generated on a static basis, where the actual explanatory variables are used to generate expected values for forecast variable in the following month. This meant that in total, twelve forecasts were generated for each month in the year 2004.

The forecasted values for each of the respective MIDAS models were then compared with a static forecast of a simple autoregressive model for the rate of inflation that used the same variables. This comparison was based on the calculation of the squared forecast error, the root mean squared error, and the mean absolute error. The calculation of these statistics is given as;

$$SFE = (\hat{y}_t - y_t)^2$$

$$RMSE = \sqrt{\sum_{t=T+1}^{T+h} (\hat{y}_t - y_t)^2 / h}$$

$$MAE = \sum_{t=T+1}^{T+h} |\hat{y}_t - y_t| / h$$

#### 6.1.1. MIDAS EXPONENTIAL ALMON LAG FORECAST MODEL

The results of the forecast statistics for the MIDAS Exponential Almon lag model are contained in Table 7. The statistic for the Mean Absolute Error suggests that the MIDAS model would produce a slightly better forecast than the DL model, however, the Root Mean Squared Errors suggests that the MIDAS model is not able to improve upon the forecast of a simple DL model. In addition, the graphs that are included as Figure 19 and 20 suggest that since the forecasting errors of the two models are highly correlated, the inclusion of high frequency data does not improve upon the results of the traditional DL model.

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	RMSE	MAE
MIDAS Exponential Almon model	0.0028732	0.002285
Distributed lag model	0.0028054	0.0023571

Table 7 – Forecasting standard errors

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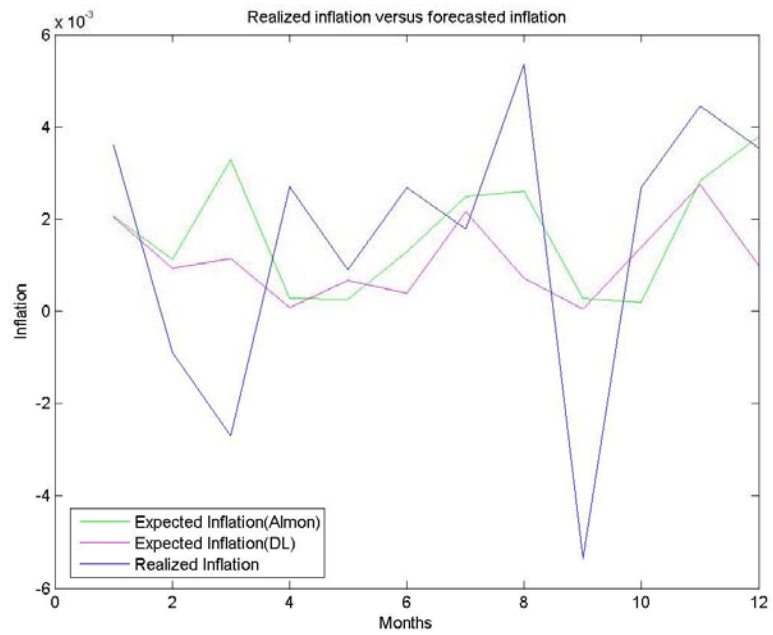


Figure 19 – MIDAS Almon lag and Distributed lag forecasts

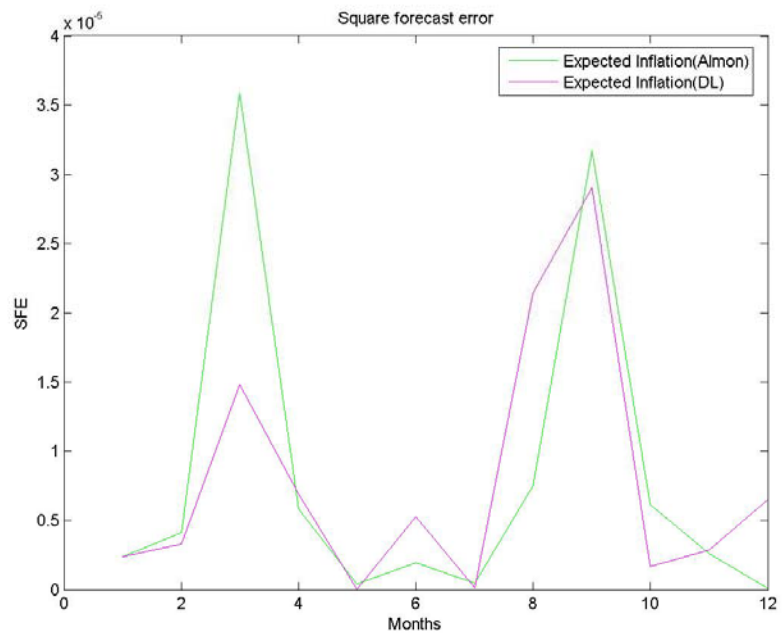


Figure 20 – MIDAS Almon lag and Distributed lag squared forecast errors

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### 6.1.2. MIDAS BETA LAG FORECAST MODEL

The results from the MIDAS Beta lag forecasting model have been contained in Table 8. Once again, these statistics suggest that whilst the MIDAS model produces a lower Mean Absolute Error it has a higher Root Mean Squared Error. In addition, the graphs included as Figure 21 and 22 suggest that the forecasting errors are still highly correlated. Hence, we are unable to conclude that the inclusion of high frequency data significantly improves upon the performance of traditional models that use asset prices that are sampled at a lower frequency to forecast inflation.

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	RMSE	MAE
MIDAS Beta lag model	0.0029382	0.0023839
Distributed lag model	0.0028916	0.0024602

Table 8 – Forecasting standard errors

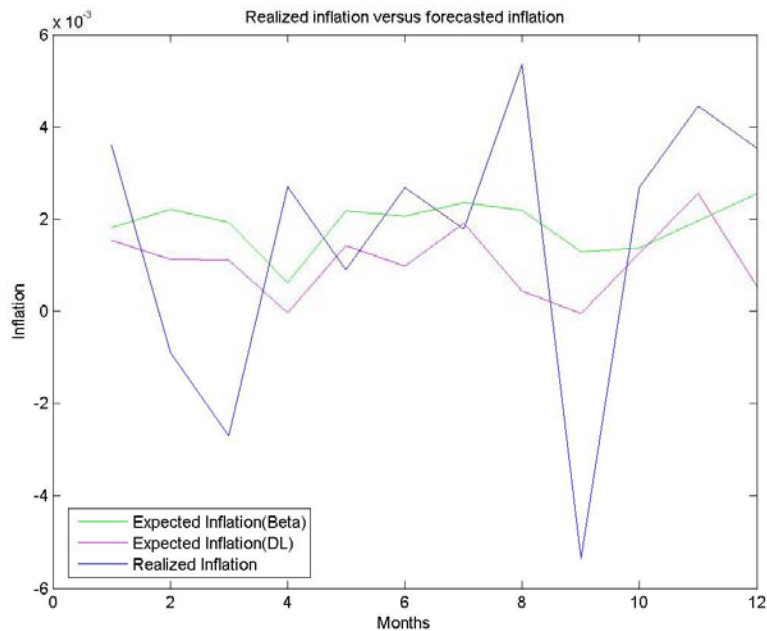


Figure 21 – MIDAS Beta lag and Distributed lag forecasts

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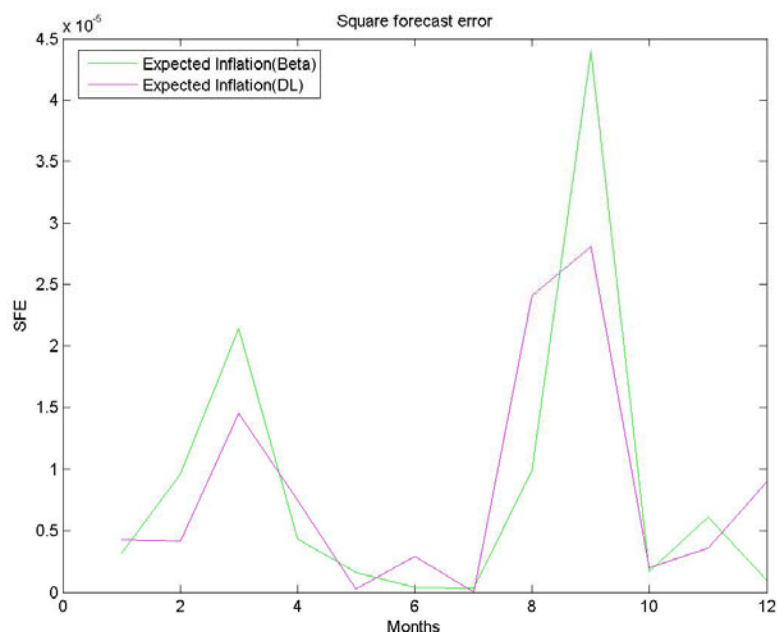


Figure 22 – MIDAS Beta lag and Distributed lag squared forecast errors

### 6.1.3. COMBINED FORECASTING OF SEPARATE EXPLANATORY VARIABLES

Stock and Watson (1999) have suggested that when forecasting the rate of inflation, it is possible to improve upon the results by combining the forecasts that are generated from several single explanatory variable models. This methodology, which utilizes postulates from dynamic factor analysis has been employed below, where forecasts for each of the optimal lag length models were combined according to weights that are determined by the regressions themselves. Therefore,

$$\bar{f}_t = \sum_{i=1}^n \omega_i f_{i,t} \quad \text{and,} \quad \omega_i = r_i^2 / \sum r^2$$

where  $\bar{f}_t$  is the predicted value of the combined forecast and the weight that is applied to each variable is dependent on the relative explanatory power of the variable.

#### 6.1.3.1. COMBINED FORECAST FOR THE MIDAS EXPONENTIAL ALMON LAG MODEL

The results from the combined MIDAS Exponential Almon lag model that are contained in Table 9 seem to suggest that the DL model outperforms the MIDAS model in terms of both the Mean Absolute Error and the Root Mean Squared Error. However, the graph of the squared forecast errors would imply that the errors that are produced by these models are highly correlated. In addition, it is also worth noting that the results from the combined forecast are worse than the results that were produced above, which does not support the earlier finding of Stock and Watson (1999).

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	RMSE	MAE
MIDAS Exponential Almon model	0.0031328	0.0028331
Distributed lag model	0.0031216	0.0027796

Table 9 – Combined forecasting standard errors

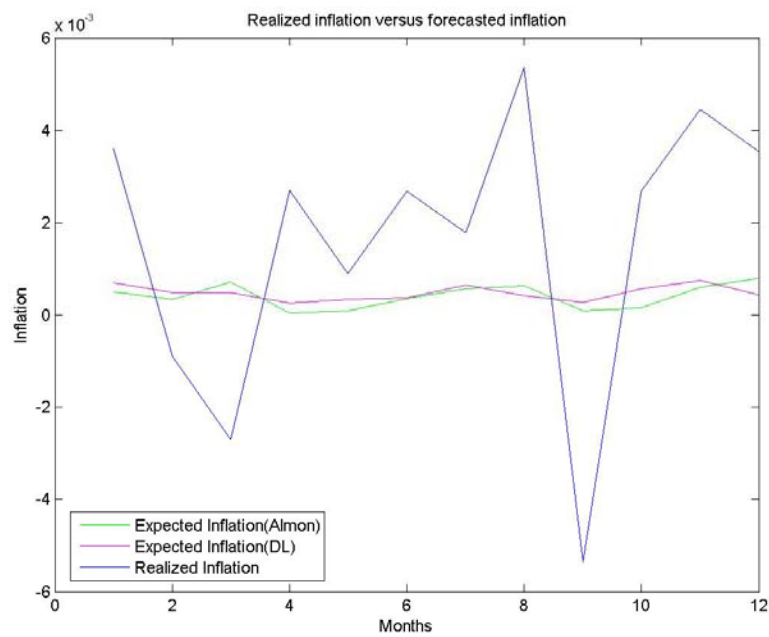


Figure 23 – Combined MIDAS Exponential Almon lag and Distributed lag forecasts

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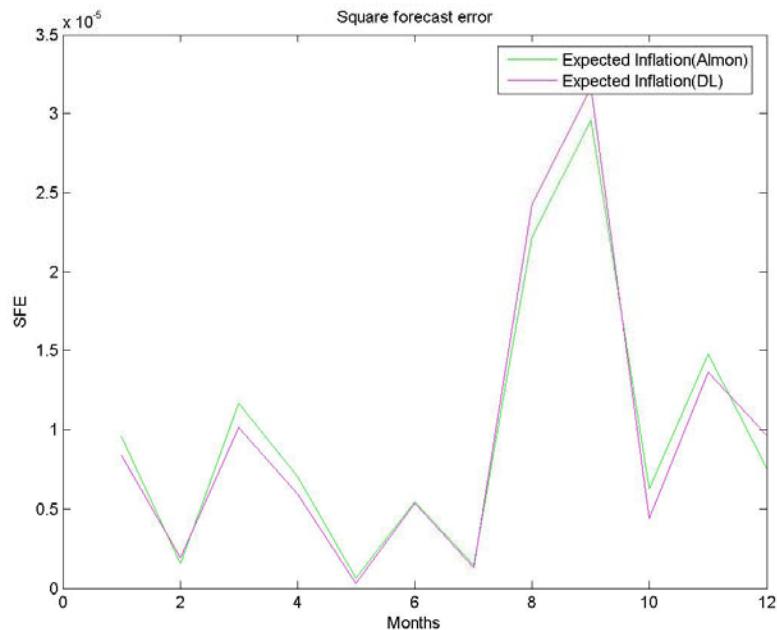


Figure 24 – Combined MIDAS Exponential Almon lag and Distributed lag squared forecast errors

### 6.1.3.2. COMBINED FORECAST FOR THE MIDAS BETA LAG MODEL

The combined statistics for the MIDAS Beta lag model that have been contained in Table 10, seem to suggest that the DL model outperforms the MIDAS model once again in terms of both the Mean Absolute Error and the Root Mean Squared Error. In addition, the graph of the squared forecast errors also seems to suggest that the forecasting errors of these models remain highly correlated. Furthermore, it is also noted that the results from the combined forecasts are also worse than the results that were produced by the above Beta lag models.

	RMSE	MAE
MIDAS Beta lag model	0.0031022	0.0027389
Distributed lag model	0.0030687	0.0026562

Table 10 – Combined forecasting standard errors

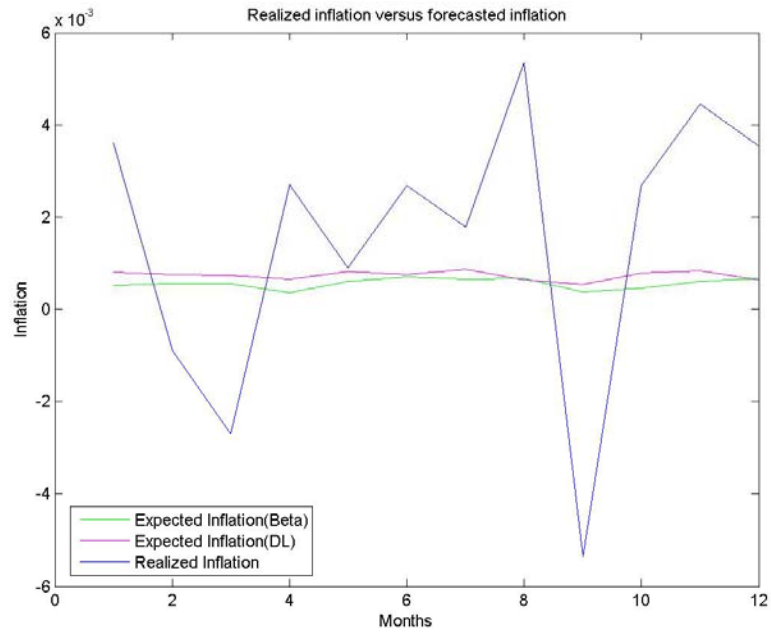


Figure 25 – Combined MIDAS Beta lag and Distributed lag forecasts

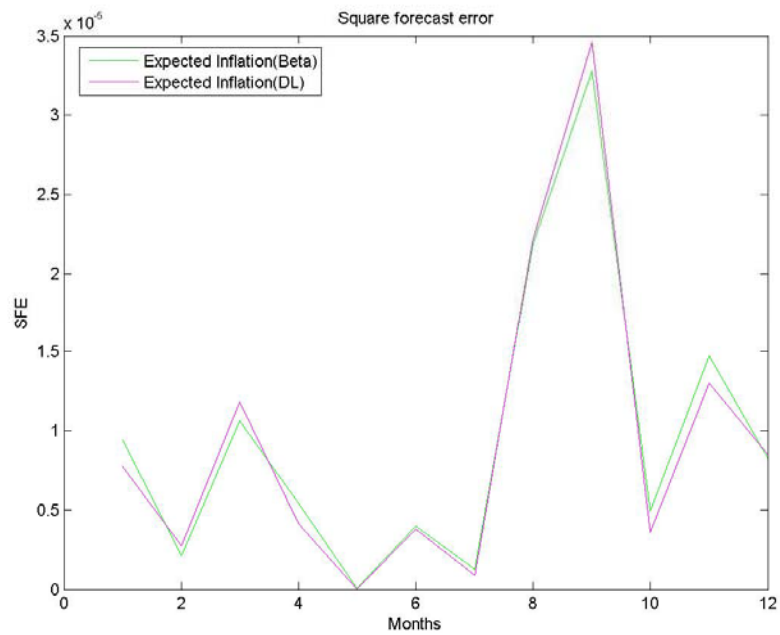


Figure 26 – Combined MIDAS Beta lag and Distributed lag squared forecast errors

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## 7. CONCLUSION

This paper provided an explanation of the general framework of the MIDAS model. It included a simple Monte Carlo analysis to show that the model struggles to derive accurate standard errors for the *theta* parameters. The model was then used to examine the relationship between the rate of inflation and asset prices, before an attempt was made to use it to forecast the rate of inflation.

Although the MIDAS model's ability to capture the additional information contained in high-frequency data is highly desirable, the problems surrounding the shape of the Exponential Almon lag polynomial may hinder the further application of this particular model. In addition, whilst the Beta lag model appears to generate more acceptable weighting functions, further investigation is needed to determine whether the weighting functions are produced from the estimates of *theta* parameters that derived from a point that represents a local minima.

With regards to the performance of the multiple explanatory variable MIDAS models, it has been shown that when forecasting the rate of inflation, the use of high frequency asset price data does not improve upon the results of traditional models that use aggregated data. This would suggest that if the application of these models was to continue, it would most probably be used to describe (or forecast) a relationship between other time series.

In addition, the out-of-sample forecasting results would also suggest that the relationship that has been described by this regression lacks a solid foundation as it has incorporated a significant amount of noise into the regression process, which has made the model somewhat unstable.

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