Resampled Mean-Variance Optimisation and the Dynamic Nature of Markets

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ABSTRACT

In this paper an approach to portfolio asset allocations is formulated within the South African domestic equity market and the diversification of assets across global markets, specifically the U.S.A. This includes the integration of a mean-variance optimiser using resampled data inputs, the mean reversion of markets, passive investment management, the selection of appropriate equity asset classes, and the use of both calendar and contingent rebalancing techniques to ensure that the portfolio remains aligned with the dynamic nature of equity markets. In this regard asset class weightings within a portfolio are measured periodically for optimality.

The research methodology constructs resampled efficient portfolios, using passive investment instruments only, and comparison is made to an equal weighted, naïve control portfolio. A unique aspect of the research is solving the issue of multiple market integration where the domestic markets are comprised of multiple asset classes, the introduction of a rebalancing methodology that examines a portfolio for optimality as well as working through the complexities of using resampled mean-variance optimisation.

The results of the research manifest unambiguous results in favour of resampled portfolios. Data resampling does seem to produce stable portfolio results that are effective at capturing a higher proportion of future returns than a naïve investment portfolio. Furthermore, the rebalancing process does provide a level of adjustment to the asset allocation to ensure optimality.

INTRODUCTION

Classic mean-variance optimisation is based on the pioneering work of Harry Markowitz (Michaud, 2004, p. 1), where the objective is to seek out portfolios that optimally diversify risk without reducing return, and to assist in the construction of an efficient portfolio.

Unfortunately the noble objectives of mean-variance optimisation fail to manifest in reality. This failure is due to the instability of outcomes as a result of data imperfections, thereby thwarting users' efforts (Black and Litterman, 1992, p. 28). As a result investment analysts have been quick to conclude that mean-variance optimisation is flawed, and without merit.

Considering that the purpose of appropriate investment advice is to improve a portfolio's performance in terms of maximising return for a given level of risk it seems counter-intuitive that investment analysts have largely ignored mean-variance optimisation. Michaud (2002, p. 2) suggests that the lack of investment value produced by a mean-variance optimiser could be the only rational explanation for the lack of adherence to modern portfolio theory principles by the investment community, which is in stark contrast to the espousals laid down in finance textbooks.

Michaud (2002, p. 3) suggests a solution to the mean-variance optimisation impasse, and this is in the form of resampling the data inputs, by means of a Monte Carlo simulation procedure, to adjust for data uncertainty. Of course real world solutions are not always that simple. Michaud does not address application issues[1] such as computational complexity, the appropriate selection of a resampling methodology and multiple market integration when each market is made up of multiple assets. Furthermore integral to the process of portfolio optimisation is the selection of appropriate asset classes, markets and portfolio rebalancing.

The study therefore is a report culminating from the quantitative study of equity portfolio optimisation, using resampling techniques, as well as the integration of multiple assets and markets, and the establishment of a rebalancing methodology to ensure that portfolios remain approximately efficient.

[1] The Michaud resampling methodology is a patented process and perhaps the lack of clarity is as a result thereof.

Throughout the various methodologies are examined with alternatives sought to reduce the mathematical complexity.

STUDY ASSUMPTIONS

1. Assets are restricted to equity alternatives.

2. Assets are selected from index tracking alternatives.

3. Investment holding periods are at least 20 years.

4. Investors seek to maximise their South African Rand outcomes.

5. Assets are allocated between the South African and U.S. markets.

6. Resampled data is determined using a historical data time series.

PREVIOUS RESEARCH

ASSET ALLOCATION

"Let every man divide his money into three parts, and invest a third in land, a third in business, and a third let him keep in reserve - Talmud (Circa 1200 B.C. – 500 A.D.)" (Gibson, 2000, p. 1)[2].

Diversification through the allocation of assets is nothing new. Modern espousal thereof is founded on the hypothesis established by Brinson, Hood and Beebower (1986) (Jahnke, 1997, p. 109), where it was postulated that 93.6 percent of a portfolio's variability was as a result of asset allocation decisions (Brinson, Hood and Beebower, 1986, p. 137), and that a little less than six percent of the variance could be attributed to market timing, stock selection and other active investment techniques.

[2] An example of a naïve portfolio which has an equal weighted asset allocation.

Bernstein (2002, p. 107) acknowledges that the Brinson, Hood and Beebower (1986) findings would be controversial, however postulates that it is arguably irrelevant how much of a return is determined by stock selection or market timing since these are uncontrollable variables. He advances that asset allocation is the only area that can be managed, and therefore should be the major focus of an investor's attention.

THE MEAN REVERSION OF MARKETS

It is prudent to infer that in order to construct forward looking investment portfolios, using historical data that future returns, although volatile and unknowable, should display movement around a known average, or there would be no value in using historical data or in determining a long term asset allocation strategy.

Interestingly Eugene Fama synthesised the theory of efficient markets[3] and yet in later research (Fama and French, 1988, p. 247), through the use of serial correlation tests, Fama and French suggest that there is 'mounting evidence that stock returns are predictable.'

In the short term stochastic pattern behaviour makes markets inherently unpredictable, and therefore efficient. A longer term view provides evidence of predictability (Edleson, 1993 p. 150). This predictability however is only available to longer term investors, and the exact timing of such predictability remains elusive.

Importantly, the implication is that long-term investors may enjoy the benefits of mean reversion, a view supported by Bradfield (2000, p. 5) within a South African context, with Malkiel (1999, p. 360) emphasising that mean reversion is applicable to markets and asset classes but not individual stocks.

THE MEAN-VARIANCE MODEL

One of the fundamental precepts of the research is what is known as the mean-variance model. The hypothesis, developed by Markowitz in 1952, argued that return alone would lead to absurd

[3] At any point in time the market prices are a good estimate of intrinsic value.

results (Markowitz, 2000, p. 52), and that a portfolio should be assessed by mean and standard deviation on the portfolio as a whole, as opposed to the weighted average of the mean and the standard deviation. Markowitz determined that portfolio risk[4], as measured by standard deviation, is a function of covariance (Evensky, 1997, p. 186).

The key insight from Markowitz's work is that the risk of a portfolio is usually less than the weighted average risk of the individual assets, and is the key to diversification.

Additionally Markowitz derived the 'critical line algorithm' which identifies all feasible portfolios, from a given set of assets, that minimise risk for a given level of return and maximises return for a given level of risk, which is known as the efficient frontier. To derive the efficient frontier requires three variables (Markowitz, 2000, p. 4), namely:

1. the expected return of the asset;

2. the expected standard deviation of the asset, and

3. the cross-correlation between the asset classes.

Initially the process of deriving the 'critical line' involved solving for corner portfolios along the line. These corner portfolios included the maximum return portfolio, the minimum variance portfolio and any number of portfolios in-between. Computing power is now able to derive the multitude of portfolios that make up the 'critical line', otherwise known as the efficient frontier.

This analysis of risk and return became known as mean-variance analysis, and later the hypothesis, together with the capital asset pricing model, became known as modern portfolio theory (Kirzner, 2000, p. 14).

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[4] $$\sigma Port = \sqrt{\sum_{i=1}^{N} W_i^2 \sigma_i^2 + \sum_{i=1}^{N} \sum_{\substack{j=1 \\ i \neq j}}^{N} W_i W_j Cov_{ij}}$$

σPort	= standard deviation of the portfolio
W_i	= weight of the individual assets in the portfolio
σ_i	= standard deviation of asset i
COV_{ij}	= covariance between returns for assets i and j, where $COV_{ij} = \sigma_i \times \sigma_j \times r_{ij}$
r_{ij}	= correlation coefficient for assets I and j

The efficient frontier is an upward-sloping curve, which reflects the key to diversification. The fact that the curve is to the left of a straight line indicates the benefit due to diversification, and is the contribution made by Markowitz based on the cross-correlation of the assets. The magnitude of the benefit due to diversification, ceteris parabis, is due to the covariance coefficient, which is derived using the correlation between assets. The lower the correlation, the higher the diversification benefit.

Given these characteristics it is intuitive to deduce that the middle portfolio on an efficient frontier should manifest a Sharpe Ratio[5] greater than either of the two extreme portfolios.

MEAN-VARIANCE OPTIMISER

With the advent of computer power, the ‘critical line algorithm’ (Kaplan, 1998, p. 2) was incorporated into a software application, known as a mean-variance optimiser (Bernstein, 2000, p. 64 - 65). Therefore, the mean-variance analysis process, by means of a mean-variance optimiser, solves the asset allocations for a given portfolio found on the efficient frontier.

It is prudent to note that the application of mean-variance analysis is not dependant on a mean-variance optimiser. The mean-variance optimiser acts as a tool to reduce the computational complexity and the propensity to commit errors in the computational process.

MEAN-VARIANCE OPTIMISER SHORTCOMINGS

‘… the most important limitations to MV *(mean-variance)* optimisation are instability and ambiguity’ (Michaud, 1998, p. 3).

Application of the mean-variance analysis process, through the use of a mean-variance optimiser, is not without serious shortcomings however, as emphasised by the above quote. The process displays considerable sensitivity to changes in the data inputs, which significantly alters

[5] SR = $\frac{(r_p - r_f)}{\sigma_p}$ where: $(r_p - r_f)$ = return in excess of the risk free rate and σ_p = standard deviation of the portfolio.

the asset allocations within a portfolio (Bernstein, 2000, p. 69), and thus has a tendency to favour assets with a recent history of high returns.

Lummer, Riepe and Siegel (1994, p. 3) state that, if the data inputs were free of estimation error, an optimiser, or optimisation, would be guaranteed to find the optimal portfolio asset allocations, and this is the imperative, since because the data inputs are estimates, typically based on historical performance, they cannot be devoid of error. Of course this is absurd, and the pursuit of estimation error free data is equally so since the future is unknown and unknowable.

Michaud (1998, p. xiii) postulates that a primary shortcoming in the use of a mean-variance optimiser is that there is a relatively ‘low level of analytical sophistication’, or mean-variance optimiser competence, in the culture of institutional investors, which is a result of the lack of a statistical understanding of how the optimiser utilises data inputs. Evensky (1997, p. 230) suggests similarly that users’ should have ‘adequate knowledge’ to correctly assess the use of a mean-variance optimiser, and its acceptance or rejection should be based on a rational and prudent analytical process. The incorrect use of data often leads to portfolios having little investment value, where a naïve portfolio is often closer to an optimal portfolio determined by using mean-variance optimisation (Michaud, 1998, p. 3).

BEYOND THE SHORTCOMINGS

Although there are many mean-variance optimiser shortcomings, the limitations largely arise as a result of viewing a mean-variance optimiser as providing an absolute answer. This view was asserted by Michaud (1998, p. 40) who postulated that a paradigm shift could result in new procedures that can reduce or eliminate many of the deficiencies.

Evensky (1997, p. 233) concurs with Michaud (1998, p. 23) by proclaiming that the knowledgeable use of a mean-variance optimiser is superior to any alternative, and that although a mean-variance optimiser has tremendous shortcomings there is a need to pursue all efforts to rationally determine asset allocations, otherwise the alternative is solely dependant on ‘judgement and intuition’ (Evensky, 1997, p. 237).

RESAMPLED MEAN-VARIANCE OPTIMISATION

In order to overcome the issue of ‘absolutes’ (Evensky 1997, p. 230) Nawrocki (n.d., p. 1) suggests that instead of seeking the ‘very best solution’ using a set of data inputs, it is better to seek out ‘an approximately good solution’.

Michaud (1998, p. 62) suggests that data input resampling leads to asset allocations that are more robust[6] and intuitive relative to classic mean-variance analysis using raw historical data. In this regard Markowitz and Usmen (2003) conceded that resampled mean-variance analysis to be superior in 10 out of 10 trials.

Resampling[7] is a process based on a stochastic simulation procedure where resampled data inputs are derived stochastically using the original historical data (New Frontier Advisors, 2001, p. 7). These stochastically derived inputs are, in turn, used as inputs into the mean-variance optimiser. This procedure is repeated numerous times, with the resultant statistically associated resampled efficient frontier asset allocations being averaged to arrive at the resampled efficient portfolio (New Frontier Advisors, 2001, p. 14). Given the level of uncertainty inherent in determining inputs, the resampling process leads to many alternative outcomes based on the original inputs. The resampled efficient frontiers are statistically associated since they are associated by means of the original set of data inputs, used in the stochastic simulation process. The association of resampled portfolios is due to their positioning on the efficient frontier. All minimum variance, middle and maximum return portfolios are, by definition, respectively associated (Michaud, 1998, p. 46).

ASSET CLASS DETERMINATION

Although an investor is free to derive a plethora of asset classes, the predominant research findings in this arena, albeit international, suggest that value equities, defined as being a high

[6] Numerous studies (Pawley (2004), De Waal and Bradfield (2003), Jiao (2003), Markowitz and Usmen (2003)) concur with Michaud (1998).

[7] It is assumed that Michaud (1998) derives new inputs stochastically using the variance-covariance matrix and a return vector as he states only that he uses the ‘Jobson and Korkie resampling procedure’ (Michaud, 1998, p. 37). After the simulation a new return vector and variance-covariance matrix is determined. These inputs are used to derive a resampled frontier. This process is repeated 500 times. There is no indication of the number of iterations used by the Monte Carlo simulator.

book value relative to market capitalisation, or a derivative[8] thereof, produce higher returns than growth equities, defined as being a low book value relative to market capitalisation, or a derivative thereof (Fama and French, 1992, p. 445). Coupled to this is the finding that equity size has an influence on equity returns, with small-cap equities showing a tendency to produce higher returns than larger-cap equities (Fama and French, 1992, p. 445).

In a South African context size and style are also important factors in determining asset classes (Pawley, 2004, p. 127). South African assets display similar characteristics to those of the U.S. asset classes, namely that value investing seems to deliver a return premium relative to growth investing (Pawley, 2004, p. 244). The study utilises outcomes using a combination of JSE Securities Exchange size classifications and style classifications based on the price-to-earnings ratio, using data for the period 1972 – 2002, a significantly larger dataset than previous research (Pawley, 2004, p. 128).

PORTFOLIO REBALANCING

Rebalancing is the process whereby a portfolio is rebalanced periodically in accordance with a predetermined asset allocation percentage. Since different asset classes yield different returns over time, at different levels of risk as measured by standard deviation, if a portfolio is left unrebalanced, the asset allocation percentages alter, which alters the standard deviation risk profile of the portfolio.

The primary caveat, with regards to rebalancing, is the issue of 'normality' of the asset allocations (Evensky, 1997, p. 265). Rebalancing to predetermined asset class weights is to presuppose that such weights remain optimal. In this regard attention needs to be drawn to the issue of input data errors where it is highly probable that the identified asset class weights may not be suitable for a future portfolio. In this regard a contingent rebalancing process that requires rebalancing based on the breach of a particular measure would be more desirable.

Michaud (1998, p. 41) suggests that a portfolio should be examined to determine whether rebalancing is required, thereby potentially avoiding unnecessary costs and consequences. Michaud indicates that there are many 'statistically equivalent portfolios' and as a result a

[8] Also defined as a relatively low price-to-earnings ratio.

portfolio that is consistent with mean-variance efficiency would not require revision. In this regard he proposes that a portfolio's asset allocations be examined[9] relative to the statistically equivalent efficient resampled portfolios derived during the resampling process. Should the portfolio fall within the 90 percent region, namely that the asset allocations fall within the range of 90 percent of the statistically equivalent portfolios, then resampling may not be required (New Frontier Advisors, 2001, p. 9).

Due to the inherent uncertainty of the data inputs, stochastically derived inputs may result in widely divergent resampled portfolios, which have a broad spread of asset allocations thereby including virtually all future portfolios. Given that the original inputs may not be appropriate (Evensky, 1997, p. 265), and therefore any resampled portfolios may result in a further move away from future optimum portfolios, it seems prudent to develop a distance function that measures how optimal a past resampled portfolio continues to be relative to the actual realised asset allocation, and adjust accordingly. In this regard Jahnke (1997, p. 111) indicates that asset allocations are dynamic, and consequently, by definition the data inputs should be dynamic, and therefore continuously revised.

THE INTEGRATION OF MULTIPLE ASSETS WITHIN MULTIPLE MARKETS

Of practical significance to the use of a mean-variance optimiser is the issue of multiple market integration. The literature in this regard combines markets through the use of broad market indices (Michaud, 1998, p. 15, Gibson, 2000, p. 154 – 156, Malkiel, 1999, p. 213 – 217, Bernstein, 2000, p. 46 – 53 and others). Therefore market integration has always been presented as a single asset class for a single market. Although this is useful in demonstrating the benefits of international diversification, this does not provide for the construction of an optimal portfolio when markets perhaps consist of multiple assets classes.

[9] The distance function proposed by Michaud (1998) is the tracking error variance (De Waal and Bradfield, 2003, p. 13). The process involves ranking the resampled portfolios by their TEV in descending order, applying a confidence region e.g. 90%, and then measuring the performance of the actual portfolio relative to the confidence region of the resampled portfolios. The process entails generating a new resampled frontier first, and then evaluating the previous averaged resampled portfolio thereto. The advantage is that there may be a reduction in the need to trade. The disadvantage is that the allocation amongst assets is very broad, thereby including virtually every conceivable portfolio (Jiao, 2003, p. 47), which will not necessary track the actual efficient frontier very closely. The excessive broadness may perhaps be overcome by narrowing the confidence region.

DATA

The data utilised by the study includes data for U.S. style and size asset classes as derived by Eugene Fama and Kenneth French (Ibbotson Associates, 2003, pp. 162 – 163), and data for South African style and size asset classes using a methodology established in Pawley (2004, pp. 31 – 34).

METHODOLOGY

COMPARISON METHODOLOGY

The portfolio derived using resampling techniques will be compared to an equal weighted, naïve portfolio consisting of the same asset classes.

RESAMPLED PORTFOLIO CONSTRUCTION METHODOLOGY

The identified asset classes are combined using a mean-variance optimiser. The asset classes are unconstrained and there is no lending or short-selling. The methodology applied to derive the data inputs for the mean-variance optimiser is a derivative of Michaud (1998), with adjustments where indicated.

1. Stochastically simulate 20 year returns[10].

2. The resampled data is used as inputs into the mean-variance optimiser.

[10] There are a number of different methodologies regarding the simulation process. Jobson and Korkie (1980, p. 547) simulated 60 – 100 observations 100 times, De Waal and Bradfield (2003, p. 9) simulated 10 000 observations, from which they sampled 120 months 500 times. The study allowed the Monte Carlo simulator to iterate 500 times and thereafter the simulated results were used. This eliminated many outliers in an attempt to reduce the problem of 'lucky draws' (Jiao, 2003, p. 43), whereby too many excessive outliers may result in skewing the averaging process. Whether this results in sub-optimal performance relative to the previous methodologies, or whether the 500 iterations needs to be revisited is the subject of further research. It must be said that it is not immediately apparent from Michaud's (1998) work how he Monte Carlo simulated the data. The study did not make use of a mean vector or variance-covariance matrix, instead the historical covariance was used and the mean and standard deviation were simulated. Numerous studies (Chan, Karceski and Lakonishok (1999), Lozano and Seppey (2001), Merton (1980), Schwartz (2000), Chopra and Ziemba (1993) as cited in Yang (2005)) indicate variance and covariance are more predictable, importantly Hirschberger, Qi and Steuer (2004) indicate that beyond a variance-covariance matrix of 10 assets it is virtually impossible to generate a valid matrix.

3. Compute the efficient frontiers.

4. Steps 1 – 3 are repeated[11] numerous times.

5. The efficient frontiers are divided[12] and ranked as minimum, middle and maximum portfolios, which reflect asset class weightings.

6. The respective portfolio weightings are averaged.

7. The resampled efficient portfolio is derived as the averaged middle portfolio.

MULTIPLE ASSET INTEGRATION METHODOLOGY

Allocating assets across multiple asset classes implies diversification. Since the objective of diversification is to eliminate any risk that is diversifiable, and therefore not rewarded, the methodology regarding the integration of multiple markets is set out below.

1. Solve for the intra-market portfolios first.

2. Apply the derived asset class weightings to historical data to calculate the respective market portfolio returns.

3. Calculate the cross-correlations between the market portfolios.

4. Adjust the foreign market portfolios for the currency movements.

5. Apply the portfolio outcomes as inputs into the mean-variance optimiser.

[11] Blindly applying the 500 repetition principle is, at times, computationally excessive and results in an averaged portfolio that is not statistically representative. The study found that by measuring the means and standard deviations of the resampled portfolios, as well as utilising a confidence interval and standard error measure the number of repetitions can be determined with precision. The number of repetitions are inversely proportional to the time period under review, and may exceed 500 at times

[12] These divisions can consist of a number of portfolios. The study chose to rank outcomes as minimum, middle and maximum portfolios. All equivalent rankings are by definition statistically related.

6. Select the minimum variance portfolio.

REBALANCING METHODOLOGY[13]

It is counter-intuitive to continuously rebalance to the established asset allocations due to the dynamic nature of asset classes as indicated by the literature. The characteristics of, and the correlation between, asset classes may change over time, leading to different asset allocation weightings within a portfolio.

It is therefore imperative that the established asset allocations be periodically tested to ascertain that they remained optimal, in this instance once annually.

The test steps are listed below.

1. The actual efficient portfolios for the respective markets are determined for the 20 year period under review.

2. The outcomes of the annual returns for the actual efficient frontier portfolio versus the previously established resampled efficient frontier portfolio are compared using the coefficient of determination (R^2).

3. Where the coefficient of determination exceeds 99 percent[14], the previously established assets allocations are deemed to remain optimal, and therefore do not require resampling.

4. Where the coefficient of determination is below 99 percent, the previously established assets allocations are deemed to be sub-optimal, and therefore require redetermination.

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[13] See footnote 9 for Michaud (1998) approach. The study applied a different methodology since although the Michaud approach may reveal the initial resampled portfolio remains statistically significant, and therefore does not require adjustment there are three reasons for the change. Firstly, the returns on the assets within the portfolio may have altered the weights of the original resampled portfolio. This could be overcome by examining the new weights using the Michaud approach. Secondly, the study concurs with Christie (2005, p. 45), investors are more concerned about means and variances than portfolio weights, therefore the study approach is to measure resampled portfolio performance relative to the classic mean-variance portfolio. Thirdly, the Michaud (1998) method is computationally excessive considering the resampled frontier needs to be calculated first. The revised approach only requires recalculation if the existing resampled frontier breaches a predetermined level.

[14] The 99 percent threshold is heuristically determined based on repeated experiments.

5. The newly established asset allocations are used for rebalancing purposes from the ensuing year.

FINDINGS AND RECOMMENDATIONS

ASSET AND MARKET INTEGRATION OUTCOMES

By first allowing the mean-variance optimiser to select from the multiple asset classes from both markets simultaneously, with reference to Figure 1 it is noted that there is limited diversification within the individual markets, with many asset classes being ignored. This is counter-intuitive and not desirable, reflecting the instability of a mean-variance optimiser.

[Insert Figure 1 Approximately Here]

With reference to Figure 2 when solving for the intra-market first there is dramatically improved diversification within the individual markets. This can be observed by a greater inclusion of asset classes, as well as a reduced exposure to the U.S. market as reflected in Figure 3.

[Insert Figure 2 Approximately Here]

[Insert Figure 3 Approximately Here]

REBALANCING OUTCOMES

An effective, predetermined asset allocation should manifest itself retroactively by displaying a coefficient of determination (R^2) of one when measured relative to the actual optimal asset allocation. Therefore the effectiveness of any asset allocation is measured by comparing the annual returns, over a specified period, of the resampled asset allocations relative to the returns, over the same specified period, of the actual optimal asset allocations. In this way it is possible to measure the amount of return determined by the resampled asset allocation. Without applying any adjustment to the original resampled asset allocations there is a steady decline in the coefficient of determination. This tendency to decline is manifested in both markets.

Given the fact that the intra-market asset allocations appear inherently unstable, and given that the inter-market asset allocations are determined by the intra-market allocations it is not surprising that the levels of determination at the inter-market level are prone to significant changes from period to period. The implication of this is that asset class instability has the tendency to maximise the movement away from an optimal allocation at the inter-market level which may result in a significant difference in portfolio outcomes relative to the optimal solution, and is an outcome not considered by Michaud (1998). Asset class instability is therefore an imperative that requires periodic attention.

Instead of remaining with the static resampled asset allocations, by reviewing the portfolios annually and redetermining the asset allocations as required the outcomes seem to display significant improvements, as evidenced in Figure 4 where it is interesting to note that the post-redetermined inter-market coefficients of determination are significantly better than the pre-redetermined figures.

[Insert Figure 4 Approximately Here]

RESAMPLED PORTFOLIO OUTCOMES

The hypothesis that mean-variance optimisation results in an effective allocation of assets, as well as effective investment diversification by constructing more optimally diversified portfolios is tested by comparing the redetermined resampled and actual asset allocations for various periods. The outcomes are represented in Figures 5 and 6.

[Insert Figure 5 Approximately Here]

It is noted in both markets, as indicated in bold, there is a significant improvement in investment diversification through a more intuitive allocation of assets. In periods where there was no improvement in allocations the implication is that previous resampled asset allocations may have been optimally allocated. The results seem to lend support to the hypothesis that resampled allocations lead to effective asset allocations.

[Insert Figure 6 Approximately Here]

RESAMPLED VERSUS NAÏVE PORTFOLIOS

The results yield that the resampled portfolios (inclusive of redetermined[15]) are significantly better than a naïve portfolio. When the rebalanced portfolios are compared to the unrebalanced portfolios, the rebalanced portfolios are significantly better in all but one instance, namely the redetermined resampled portfolio where the unrebalanced portfolio is superior. Interestingly, although the redetermined resampled unrebalanced portfolio was supreme, the redetermined resampled rebalanced portfolio was significantly better than any of the alternative portfolios.

This finding shows that, although a rebalanced portfolio is likely to yield a superior outcome, this is not assured[16]. In this instance the impact on the rebalanced redetermined resampled portfolio was a result of the fact that the U.S. redetermined resampled intra-market asset allocation showed significant divergence from the actual asset allocation.

The findings are graphically depicted in Figure 7.

[Insert Figure 7 Approximately Here]

MARKET DYNAMISM AND ASSET ALLOCATIONS

Given that markets are inherently dynamic no established asset allocation remains optimal for every time period, and therefore requires periodic revision. This process is a combination of contingent (testing a portfolio for optimality) and calendar (adjusting a portfolio to predetermined weights) rebalancing.

The outcome of the application of these methodologies is reflected in Figure 7. Notably the redetermined portfolios manifest Sharpe Ratios in excess of both the rebalanced and unrebalanced resampled portfolios, which seems to lend support to the thesis of adjusting continuously for market dynamism.

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[15] Asset class weightings that have been adjusted using a resampling methodology when the portfolio fails the optimality test.

[16] 'So how do you arrive at the allocation that will provide the most return for the least amount of risk? You can't. But don't feel bad, because neither can anyone else.' (Bernstein, 2000, p. ix).

RECOMMENDATIONS

The purpose of the study was not to identify new or more appropriate asset classes. For this reason the choice of asset classes is not an imperative for the study. What is clearly apparent however is that the process of constructing an investment portfolio, using resampled mean-variance optimisation, and making periodic adjustments for market dynamism seems to be unambiguously superior to a naïve portfolio. Although as to the most effective methodology to be applied this requires further research.

Some form of resampled mean-variance optimisation seems prudent, and should be investigated by investors and investment analysts alike. In this regard both categories would be well served to acquire the skills necessary to effectively apply the technique.

CONCLUSION

As with any research the chosen methodology is always a contentious issue. This study is no exception. There is no holy grail in this regard; however the imperative is that any efforts to enhance portfolio returns over the long term can only be deemed as crucial since investors rarely operate within the confines of a long term investment policy, structured around a sound asset allocation framework.

Finally, in the context of the conference it would be apt to address the question “Development Perspectives – Is Africa Different?”

No, there is seemingly no application of Modern Portfolio Theory within the South African context beyond academic teachings given that “it is widely known that traditional mean-variance optimisation yields unrealistic portfolios” (De Waal and Bradfield, 2003, p. 3).

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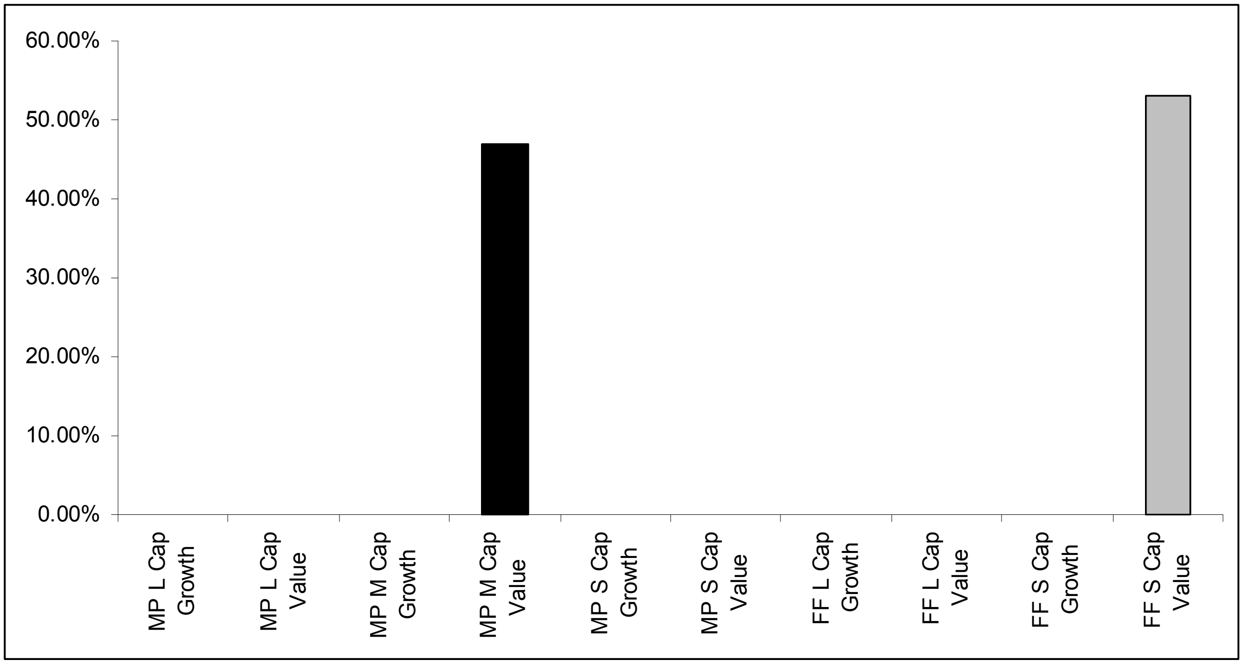
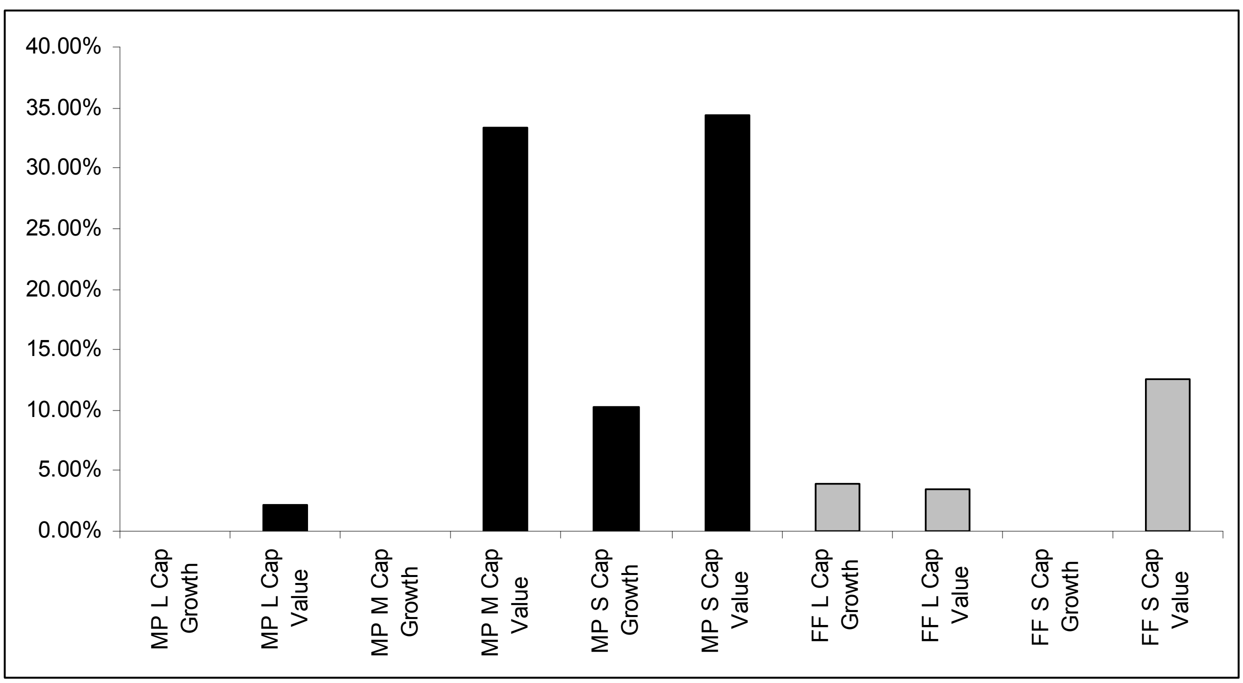
FIGURES

Figure 1 **Combined inter-market asset allocation (1973 – 1992)**



Source: Adapted from Pawley (2004, p. 231).

Figure 2 **Combined intra-market asset allocation (1973 – 1992)**



Source: Adapted from Pawley (2004, pp. 168 -169).

Figure 3 **Comparative inter-market analysis (1973 – 1992)**

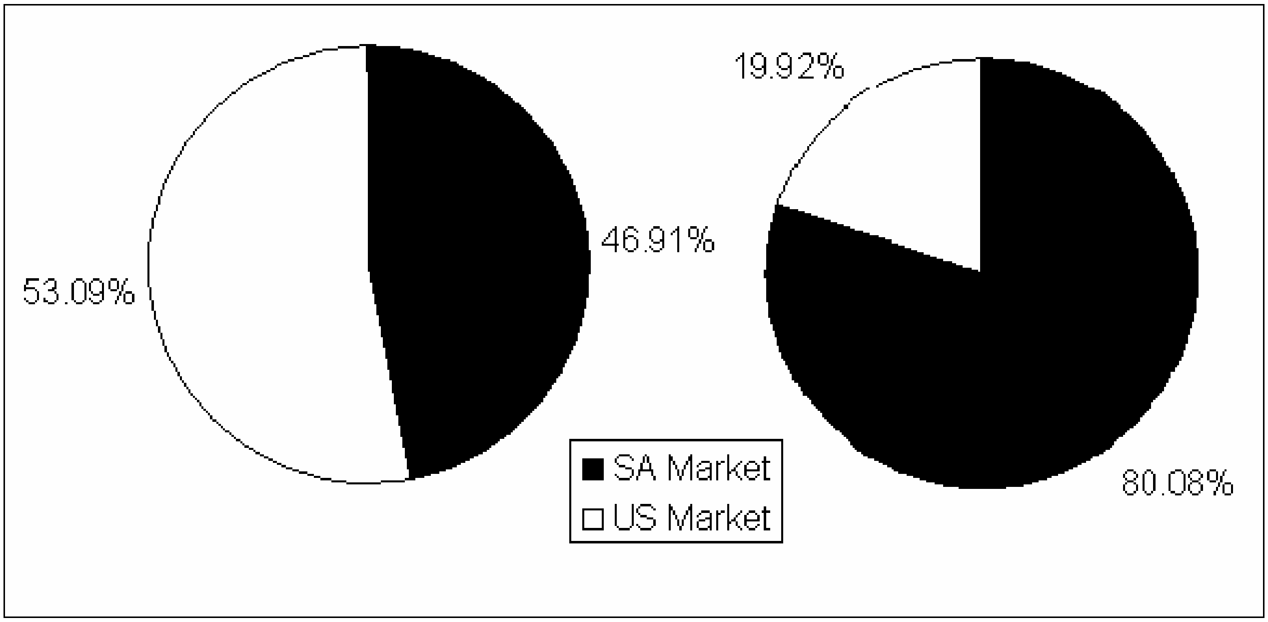
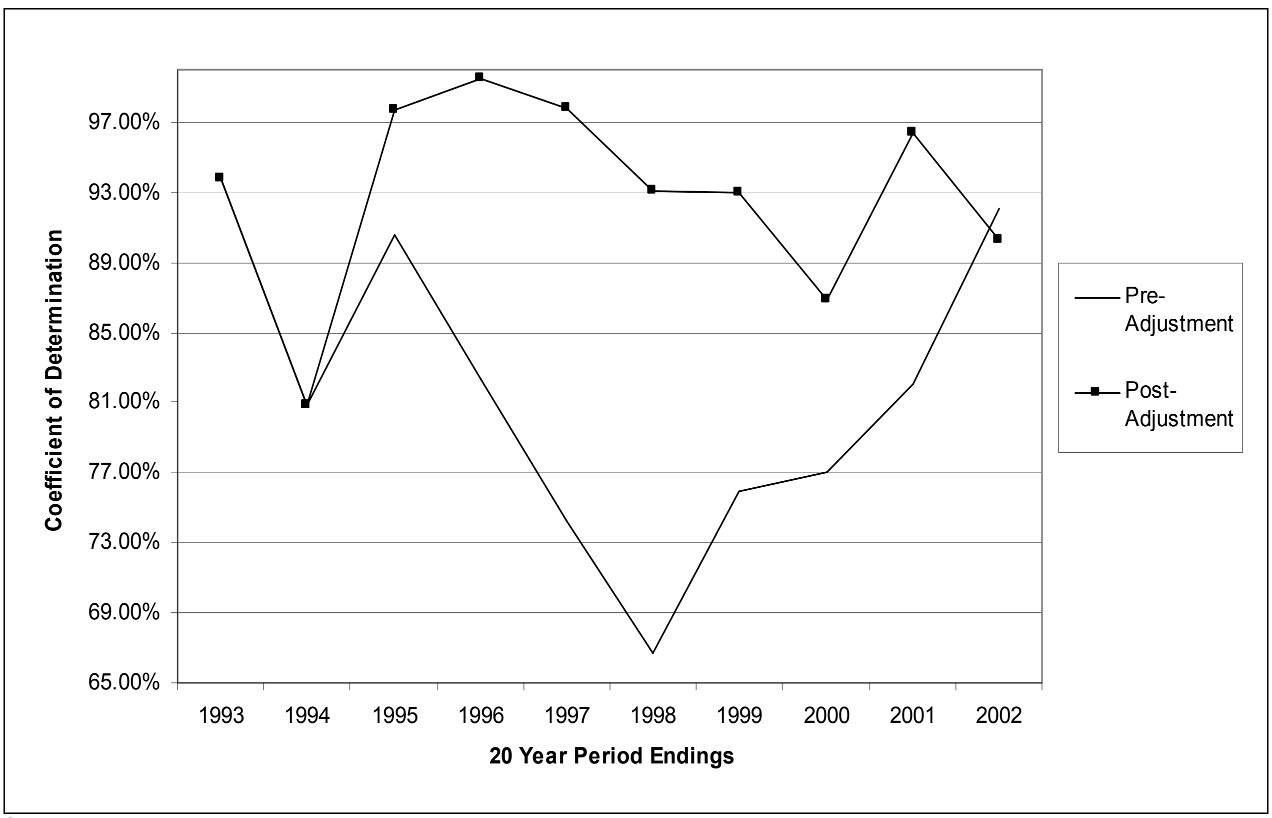
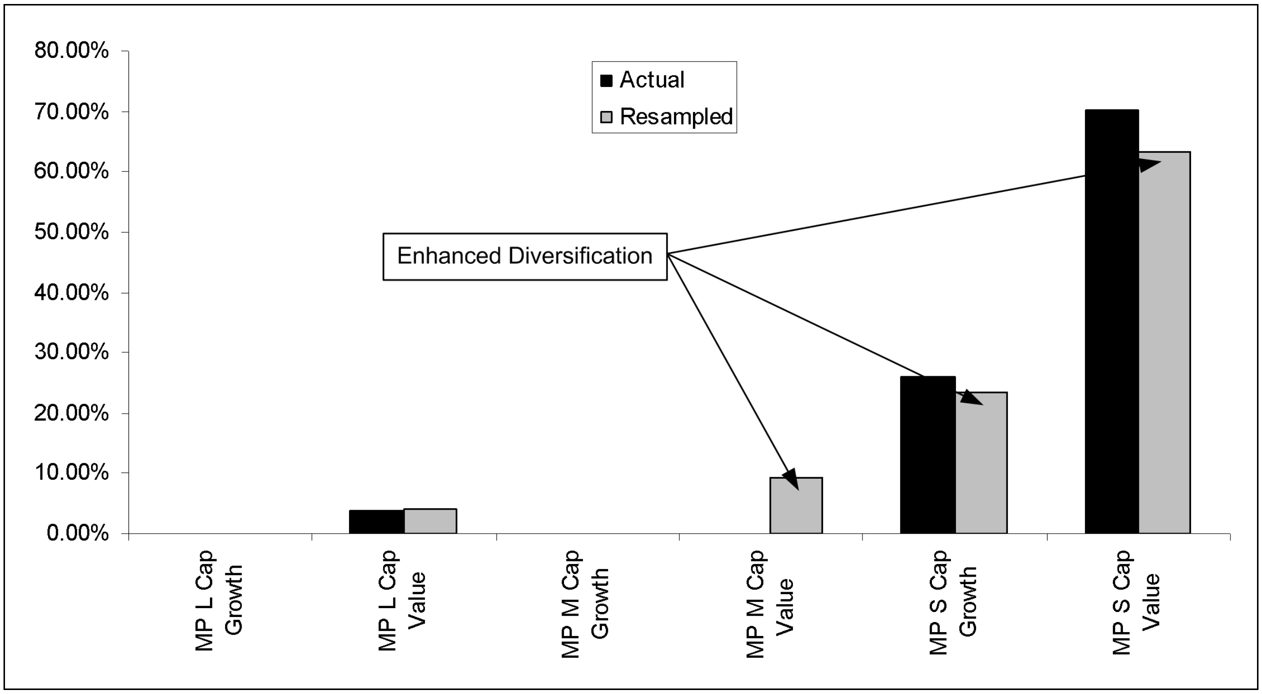


Figure 4 **Inter-market equities post-redetermination (1993 – 2002)**



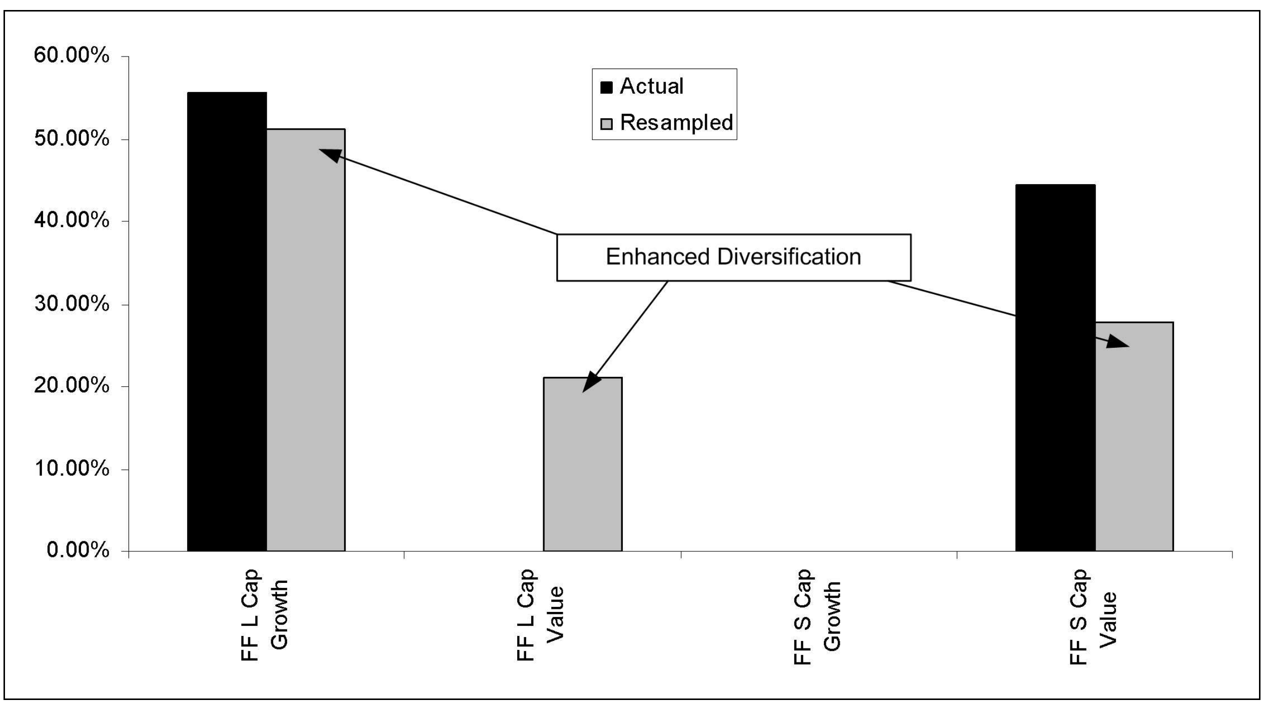
Source: Pawley (2004, p. 206).

Figure 5 South African equities (1975 – 1994)



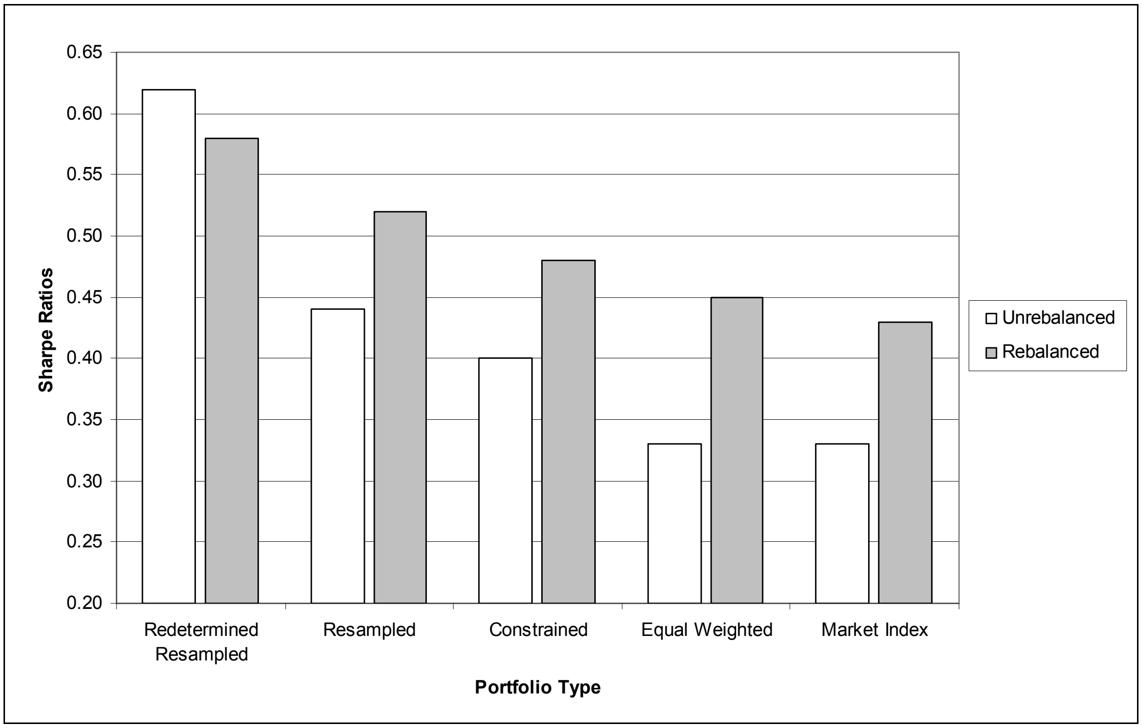
Source: Pawley (2004, p. 208).

Figure 6 U.S. equities (1980 – 1999)



Source: Pawley (2004, p. 209).

Figure 7 **Investment portfolio Sharpe ratios (1993 – 2002)**



Source: Pawley (2004, p. 212).