The distribution of exchange rate volatility of the South African currency

Prepared by

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September 2009

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[1] The views expressed are those of the author(s) and do not represent those of the South African Reserve Bank or the South African Reserve Bank's policy.
[2] This paper was prepared for the ESSA Biennial Conference 2009 of the Economic Society of South Africa, held in September 2009 at the Feather Market Convention Centre, Port Elizabeth - South Africa.

Introduction

- The fundamental issue to financial asset pricing is the underlying future asset returns volatility
- The underlying future asset returns volatility quantifies the associated risk or uncertainty
- Thus appropriate measurement of asset returns volatility is important to financial assets risk analysis
- The validity of volatility measures depend on distributional assumptions concerning the underlying asset risk - Anderson et al (1999)
- There is still uncertainty concerning the distribution that generally models returns of financial assets
- Theory makes no assertion about the distribution of financial assets returns and the consensus on which distribution is appropriate has not been reached

Introduction

- Financial assets market models have assumed that the densities of most financial assets returns are independently and identically distributed or Gaussian - Fergusson and Platen (2006)

- It is an established fact that stock and foreign exchange markets returns densities demonstrate heavier tails or are leptokurtic - Platen and Sidorowics (2007)

- They have large excess kurtosis so that they are more peaked than the Gaussian distribution suggests - Platen and Sidorowics (2007)

- Aim – to identify the distribution which characterises the returns volatility of the South African currency against the four major currencies - the US dollar, British pound, Japanese yen and the euro

- Of particular interest is whether the volatility of the South African currency returns densities are Gaussian or Student’s-t distributed

Literature reveiw

- Financial assets volatility is not directly observable such that it essentially depends on the statistical model that is applied to the assets returns data

- The stochastic process which generates financial assets returns changes over time such that these returns are typically characterised by time-varying volatility, persistence and volatility clustering

- The ARCH model – by Engle (1982) and the GARCH model – by Bollerslev (1986) and Taylor (1986)

- They are specifically designed to model these stylised facts and as such are typically used to analyse financial assets risk

- ARCH and GARCH models are commonly estimated by the method of maximum likelihood - an assumption about the conditional distribution of the error term is needed

- The common assumptions about the distribution of the error term – the Gaussian (normal) distribution, Student's-t distribution and the General Error Distribution (GED)

Literature review

- There is an immense empirical literature in search of the distribution which typically characterises financial assets returns

- A considerable amount of empirical studies concentrates on volatility distributions of stock market returns

- However, there are also studies which seek to uncover the distribution of foreign exchange returns

- Anderson et al (1999) – Used a data set comprising of ten years of high frequency (5 minute) Deutschemark and Japanese yen per US dollar returns

- Anderson et al (1999) – Found that the distributions of realised daily variances, standard deviations and covariances are skewed to the right and also leptokurtic

- Anderson et al (1999) – The distributions of logarithmic standard deviations and correlations are approximately Gaussian

Literature review

- ❖ Anderson et al (2000) – Analysed the same data set but standardised by realised volatilities of Anderson et al (1999),
- ❖ Anderson et al (2000) – found that these returns are also approximately Gaussian, the finding which they contend is consistent with theoretical prediction
- ❖ Kilic (2007) – Using six nominal daily US dollar exchange rates, estimated the GARCH and the Fractionally Integrated GARCH models with normal, Student's-t and normal inverse Gaussian (NIG) error distributions
- ❖ Kilic (2007) – Found the model with NIG errors is capable of capturing the peakedness and fat tails in conditional volatility of the exchange rate returns
- ❖ Platen and Stahl (2006), Fergusson and Platen (2006) as well as Platen and Sidorowics (2007) – Identify that the Student's-t distribution with about four degrees of freedom as an appropriate estimated distribution of diversified stock indices expressed in different currencies

Model

- The volatility of financial assets returns is not directly observable and can only be observed in the context of a model

- The GARCH model is used to quantify the South African currency exchange rates volatility

$$y_t = \theta + \varepsilon_t \qquad [1]$$

$$\sigma_t^2 = \omega + \sum_{i=1}^{p} \alpha_i \varepsilon_{t-i}^2 + \sum_{j=1}^{q} \beta_j \sigma_{t-j}^2 \qquad [2]$$

$$\alpha_i \geq 0 \text{ , } \beta_i \geq 0 \text{ and } \sum_{i=1}^{p} \alpha_i + \sum_{i=1}^{q} \beta_i < 1 \qquad [3]$$

- Equation [1] is the mean equation, while equation [2] gives the conditional variance equation

- σ_t^2 is the conditional variance, ε_{t-i}^2 is the ARCH term and σ_{t-j}^2 or the GARCH term.

Model

- These models have become standard tools in empirical macroeconomics and finance risk analysis

- They allow the determination of the presence of volatility in the objective manner by providing parsimonious approximations to volatility dynamics

- They have generally demonstrated useful properties for estimating the mean and conditional variance of volatility dynamics in a unified framework

- They are also consistent with the stylised facts in foreign exchange rate dynamics such as persistence and volatility clustering

- The GARCH type models are estimated using the method of maximum likelihood - the assumption about the distribution of the residuals is required

- The three assumptions which are usually used are Gaussian, Student's-t and the General Error Distributions

Model

- The choice of any particular error distribution only affects the covariance matrix but not the estimated parameters

- When the residuals are not conditionally normally distributed, ARCH parameter estimates remain consistent, whilst the covariance matrix is non-consistent so that the standard errors are incorrect

- The focus in this study will be on the Gaussian and the Student's-t distributions - dominate the findings in empirical studies

- Several approaches are used to determine the appropriate distributional assumption for any particular time series.

- The examination of theoretical Quartile-Quartile (QQ) plots of the residuals, the Log-likelihood values of the estimated models different distributional assumptions and the comparison of the residuals' kernel densities against a fitted theoretical distribution

- Some authors such propose evaluating the forecast performance of a particular model under different error distribution assumptions - Wilhelmsson (2006)

Data

- The data consists of nominal daily closing spot exchange rates of the South African rand against four major currency markets

- These are the US dollar (USD), British pound (GBP), the Japanese yen (JPY) and the Euro (EUR)

- The sample covers period January 01, 1999 to Dec 31, 2008, and data is sourced from the Bloomberg database

- All the exchange rates are transformed into continuously compounded returns.

- The advantage is that they have attractive statistical properties of egordicity and stationary after this transformation

- This ensures that they behave in a similar manner and that they can be modelled as a stationary process

Variables’ descriptive statistics

	EUR	GBP	JPY	USD
Mean	0.0022	0.0017	0.0000	0.0015
Std. Dev.	0.1168	0.1649	0.0011	0.0968
Skewness	1.2870	0.6255	1.2630	2.0147
Kurtosis	50.099	47.432	49.023	34.941
Observations	2648	2648	2648	2648

- ❖ The South African currency has been most volatile against the British pound and least volatilitile against the Japanese yen
- ❖ All the currencies are fat tailed compared to a normal distribution (the kurtosis exceeds 3 – implies leptokurtosis) and are positively skewed

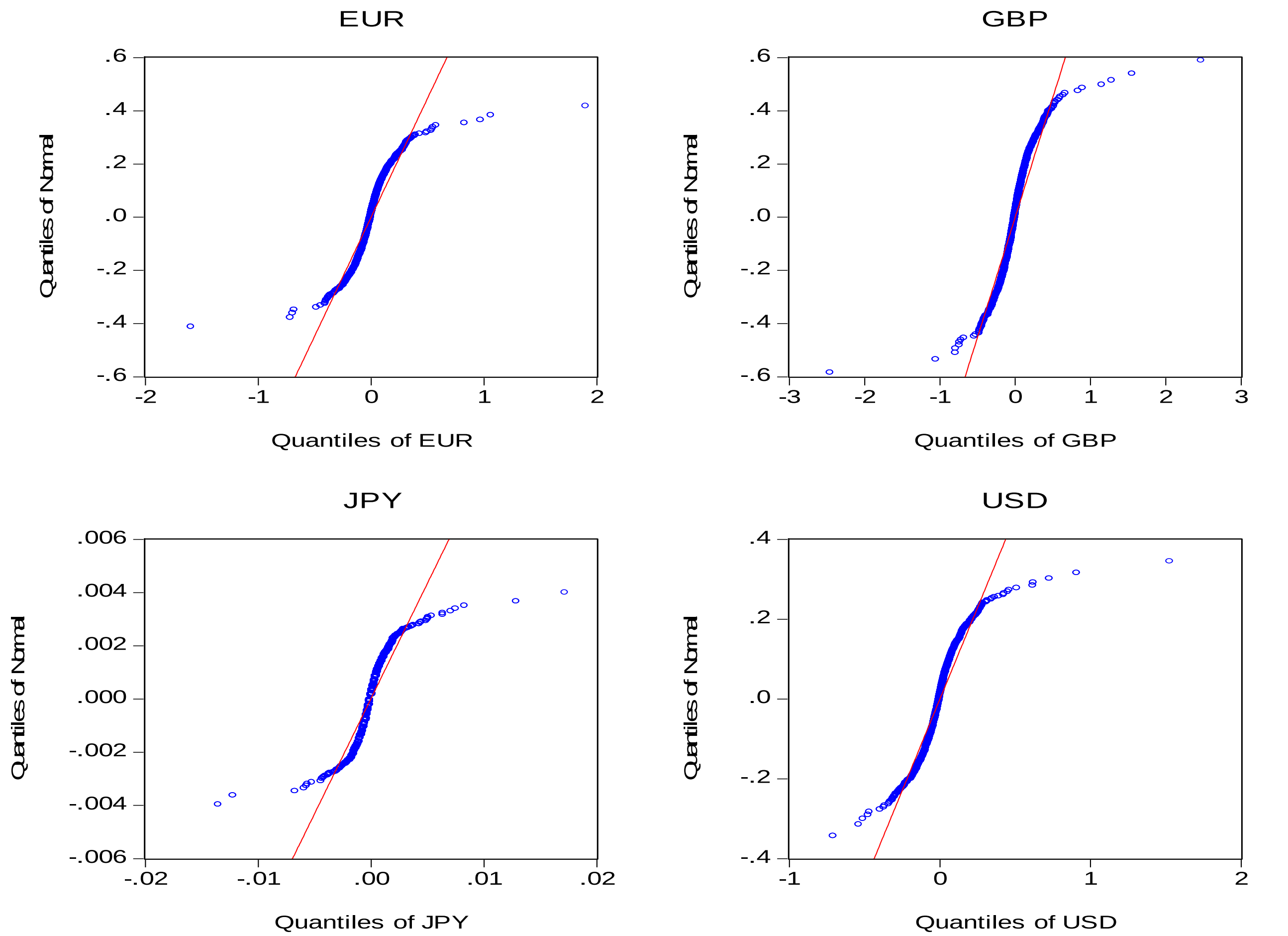
Empirical results

- ❖ The GARCH(1,1) model are used to quantify the South African currency exchange rates volatility
- ❖ Out-of-sample forecasts comparisons of 330 volatility models – no statistically significantly better forecasts than the GARCH(1,1) model – Hansen and Lunde (2001)
- ❖ Of the family of volatility models, the GARCH(1,1) model is the simplest and the most robust – Engle (2001)
- ❖ In order for the GARCH model to adequately describe volatility dynamics, it should have no statistically significant remaining ARCH in the residuals
- ❖ The ARCH-LM test results – the null hypothesis of no remaining ARCH effects in the residuals is accepted for all the estimated models
- ❖ The estimated GARCH(1,1) model results – All the ARCH and GARCH parameter estimates are highly statistically significant

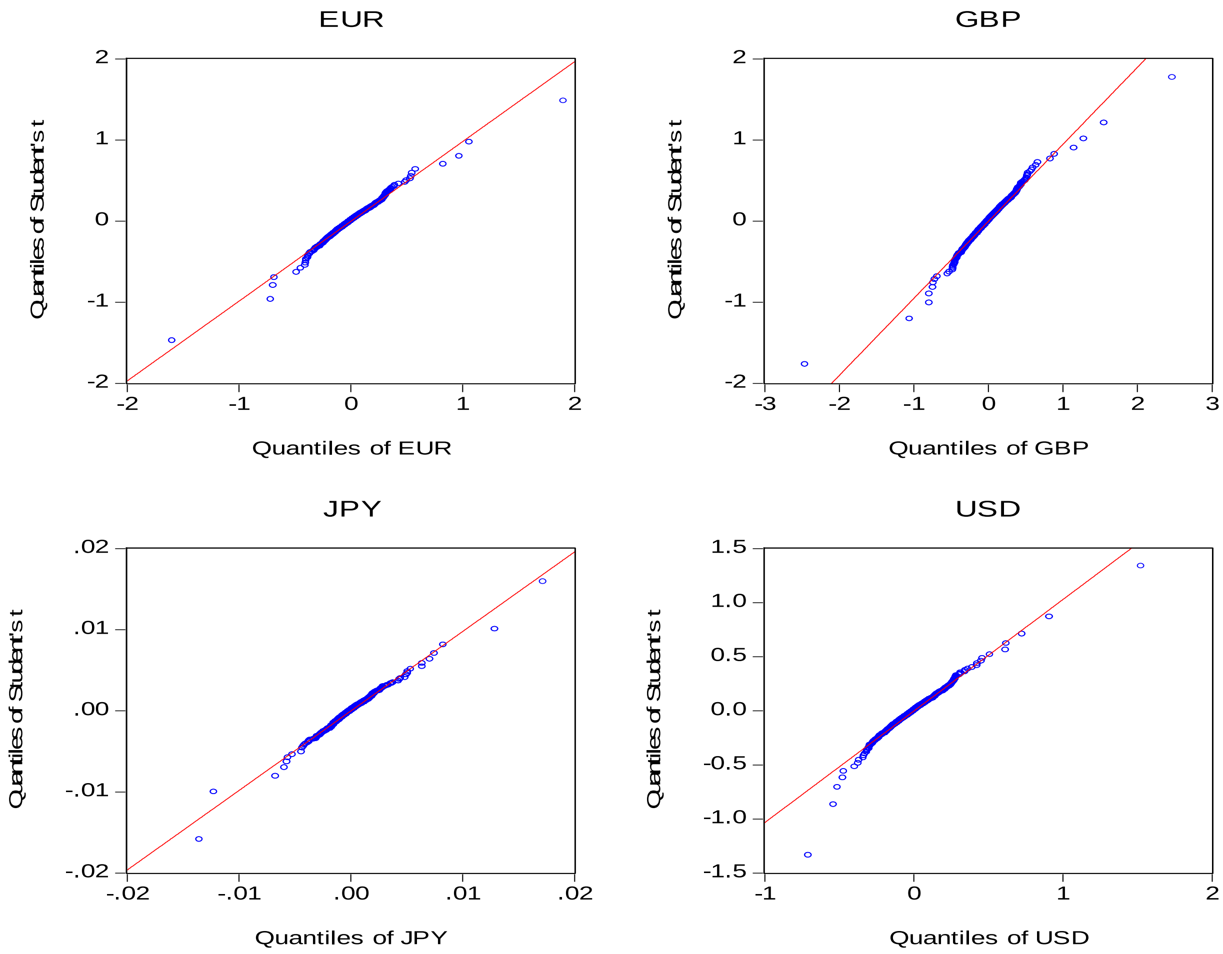
Quartile-quartile plots

- The quartile-quartile plot is graphical technique which uncovers whether a time series follow a particular distribution
- Plotting the quartiles of a particular time series against the quartiles of a specified distribution presented as a straight reference line
- If the quartiles of this particular time series lie along this reference line, the conclusion is that it follows the specified distribution
- If there is a mismatch such that there is a departure from the reference line, the conclusion is that the time series differ from the comparison distribution
- The plotted residuals depart from the normal distribution, particularly at the extremes - the large negative and positive residuals are driving the departure from normality
- The plots of the residuals assuming a student's-t distribution are better aligned to the reference line compared to the residuals assuming a normal distribution

Quartile-quartile plots assuming a normal distribution



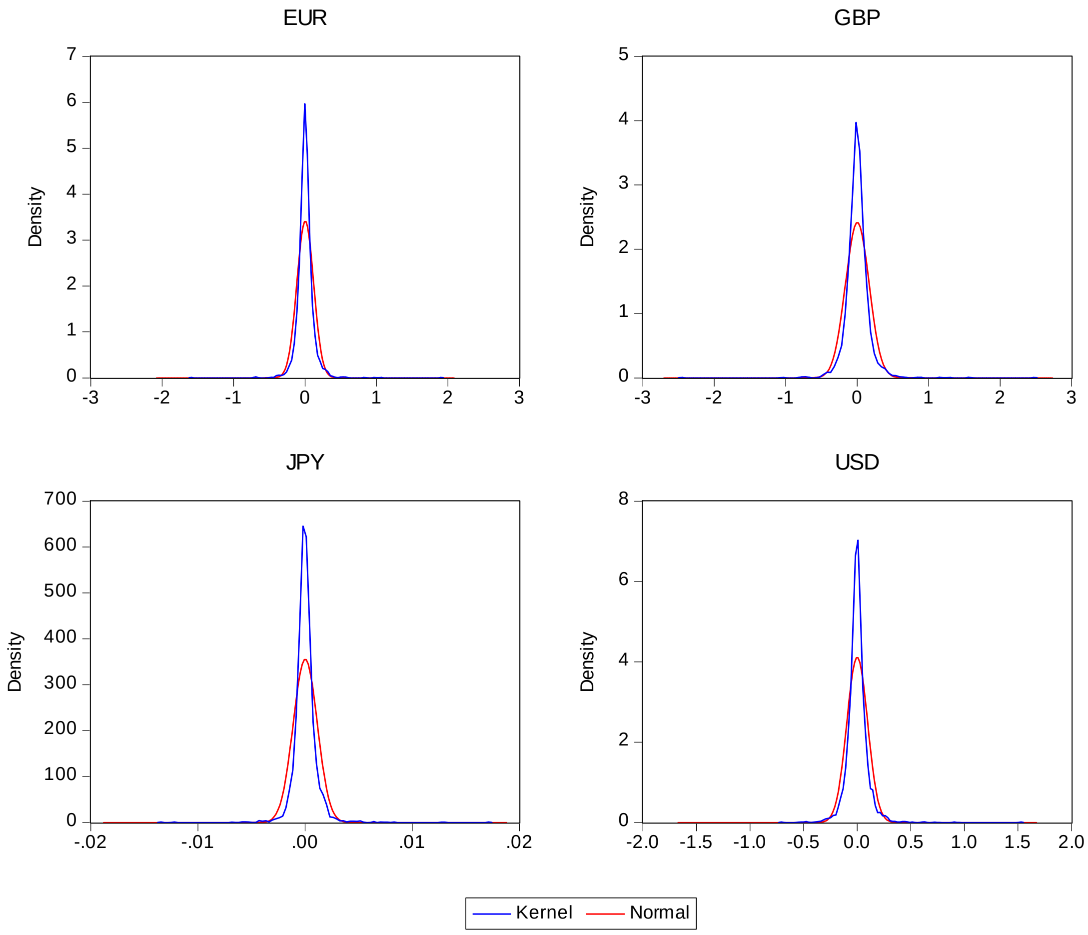
Quartile-quartile plots assuming a Student's-t distribution



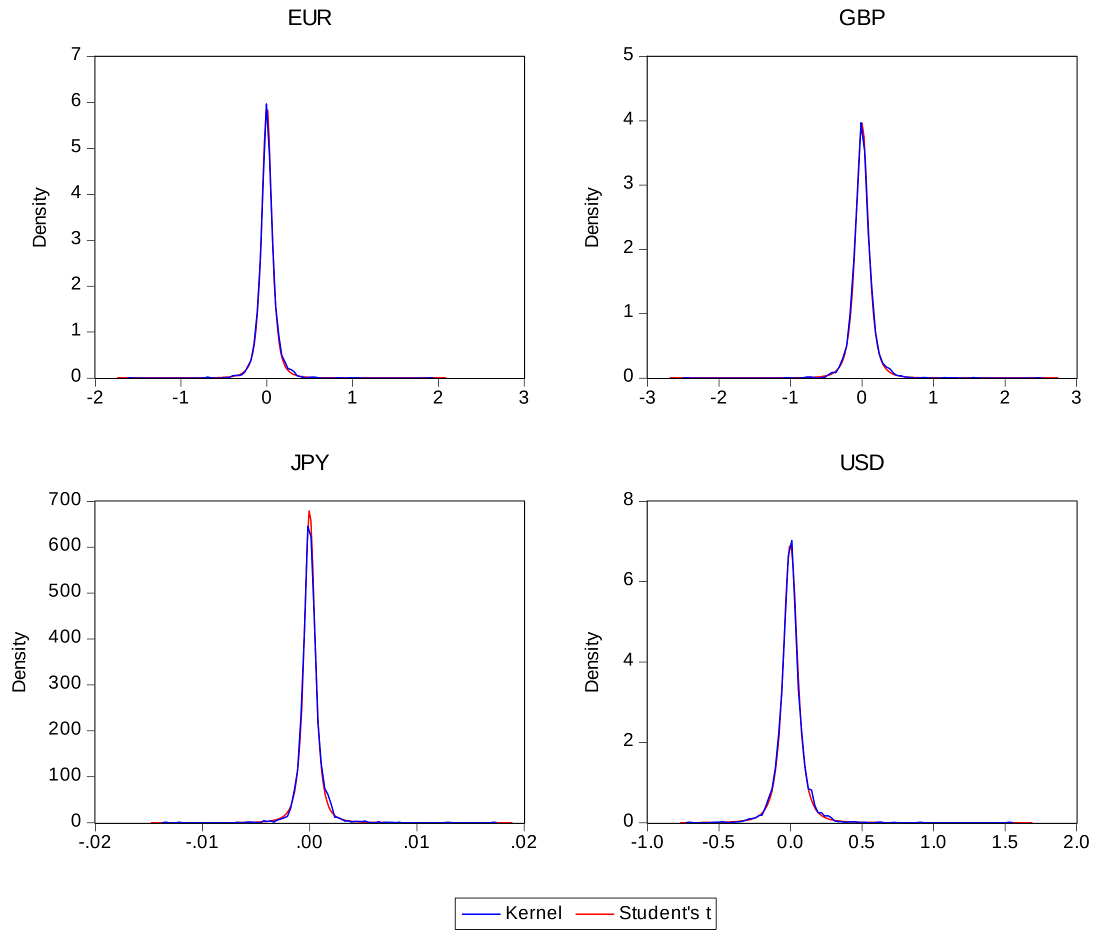
Kernel densities plots

- A kernel density plot is a smoothed density function of a histogram of a time series
- Smoothing is achieved by placing more weight on observations that are closer to the point being evaluated
- The kernel densities plots of the residuals are compared with the fitted theoretical normal and Student's-t distributions plots
- The kernel densities of the residuals are relatively higher peaked and more leptokurtic compared to the normal distribution
- The kernel densities of the residuals are more in line with the Student's-t distribution because they fit its peakedness and leptokurtosis better

Kernel density plots assuming a normal distribution



Kernel density plots assuming a Student's-t distribution



Log-Likelihood values

- In maximum likelihood estimation, the log-likelihood functions adjust the parameter estimates in the GARCH model to give the best fit
- The best model is one whose log-likelihood value is noticeably larger than the other models for any specific distributional assumption
- The log-likelihood values of the estimated model assuming that the residuals follow a normal distribution are consistently smaller
- Larger log-likelihood values are achieved under a Student’s-t distribution assumption for all the selected exchange rates returns

Log-Likelihood values

	Normal distribution	Student's-t distribution
	Log likelihood	Log likelihood
EUR	2732.66	2838.45
GBP	1739.23	1837.18
JPY	15226.00	15322.98
USD	3233.75	3312.44

Conclusion

- The aim was to identify the distribution which characterises the South African currency returns volatility against the US dollar, British pound, Japanese yen and the euro.
- The GARCH(1,1) model are used to quantify the returns volatility of the South African rand against four major currency markets over ten years
- Of particular interest is whether the South African returns densities are Gaussian or Student's-t distributed
- The theoretical Quartile-Quartile (QQ) plots of the residuals, the Log-likelihood values of the estimated models and the comparison of the residuals' kernel densities
- The Student's-t distribution is relatively a more appropriate distribution assumption for the South African currency returns compared to the Normal distribution
- For further research – more probability distributions and exchange rates should be considered

End…

Figure A1 Nominal exchange rates

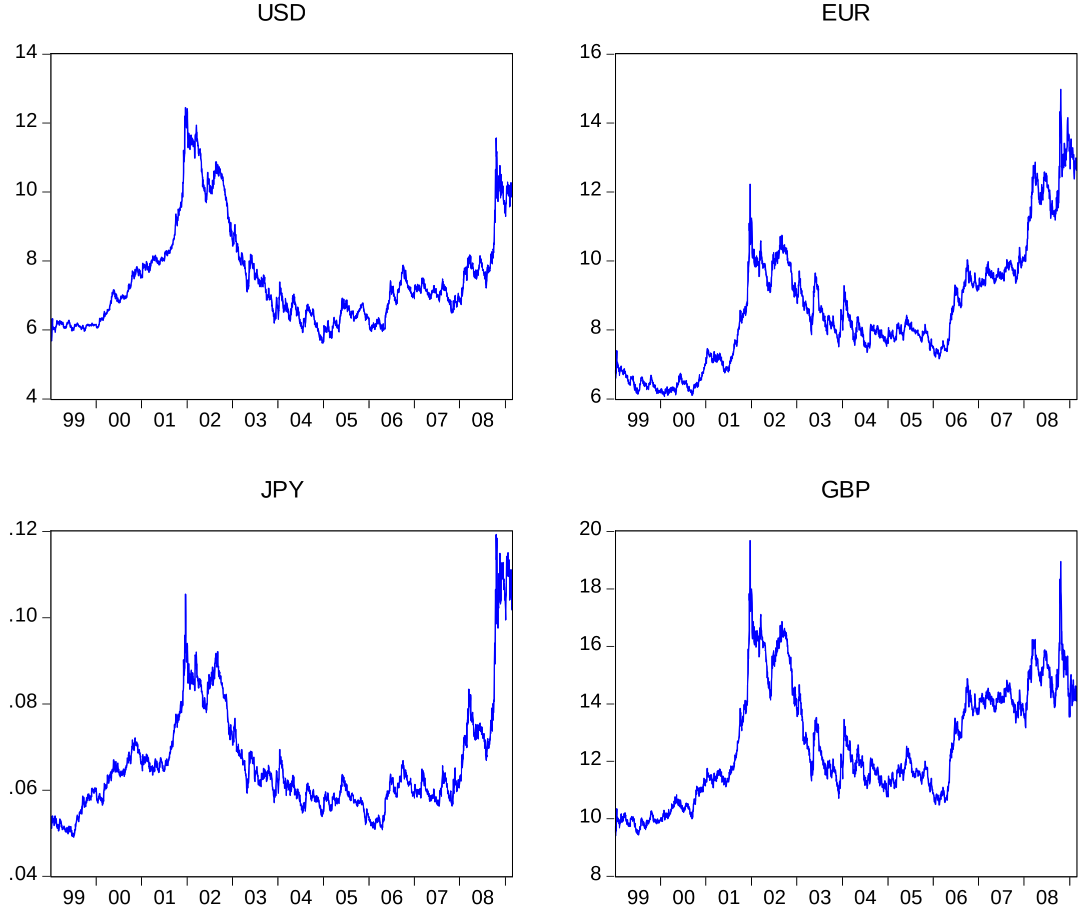


Table A1 GARCH(1,1) RESULTS

		ω	α	β
Normal distribution	**EUR**	0.0001 (5.520)	0.1169 (14.04)	0.8811 (102.4)
	GBP	0.0001 (3.920)	0.1000 (15.65)	0.9044 (155.8)
	JPY	0.0000 (7.440)	0.0953 (19.38)	0.8987 (217.4)
	USD	0.0000 (3.010)	0.1034 (18.89)	0.9065 (179.9)
Student's-t distribution	**EUR**	0.0001 (3.870)	0.1060 (7.710)	0.8843 (67.34)
	GBP	0.0003 (3.650)	0.0984 (7.960)	0.8927 (76.09)
	JPY	0.0000 (3.760)	0.0939 (7.520)	0.8944 (71.07)
	USD	0.0000 (1.860)	0.0920 (8.260)	0.9160 (101.6)