Devolution of the Fisher Equation:
Rational Appreciation to Money Illusion

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At the outset the question arises, how can a merchant be said to foresee the appreciation of money? Appreciation is a subtle conception. Few business men have any clear ideas of it. Economists disagree as to its definition, and statisticians as to its measurement.

Irving Fisher (1896, p. 35)

Genesis of the Paper

- Criticisms of the "Fisher equation" in the 1980s.
- My failed attempt to defend the Conventional Fisher Equation (CFE).
- My surprising discovery when reading *Appreciation and Interest* (1896): The Original Fisher Equation (OFE) is different from the CFE!

Three Claims

- Fisher never published an *explicit* form of the CFE.
- The OFE is a distinct equation not a transposed form of the CFE.
- Fisher's (1930) rejection of the OFE was a result of "money illusion" displacing "rational expectations" as a theory of market psychology.

Conventional Fisher Equation (CFE)

$$i = r + \pi + r\pi$$

$$\pi \equiv \frac{EP_{t+1} - P_t}{P_t}$$

Original Fisher Equation (OFE)

$$j = i + \mathrm{a} + \mathrm{ia}$$

$$\mathrm{a} \equiv \frac{E(1/P_{t+1}) - (1/P_t)}{(1/P_t)}$$

Why OFE is Different

- CFE uses value of goods (P) whereas OFE uses value of money (1/P).
- Jensen's inequality implies:
  - $1/E(P) \geq E(1/P)$
  - $a \neq -\pi$
  - $j \geq r$
- Two are equivalent if and only if:
  - Same base year is used
  - Perfect certainty of forecasts

Does It Matter?

- CFE can be considered an approximation to the OFE. But how good?
- Both the theoretical and empirical literatures indicate that the two measures of expected purchasing power (a and π) can diverge substantially.
- Bottom Line: It does matter how expectation operators (E) are used.

Did It Matter to Fisher?

- In all his works, Fisher equated rationality with expected appreciation (a) *not* expected inflation (π).
- Fisher's (1896) empirical work calculated expected appreciation of money (a) using the OFE and the difference in yields between gold bonds (j) and paper bonds (i).

Fisher's Money Illusion

- Fisher's empirical work (1896, 1930) led him to question the rationality of market expectations.
  - Market participants consistently under-predict money value.
  - Interest rates fail to take adequate account of the changing value of money.
- Fisher proposed a psychological theory of expectation formation based on "money illusion."
- Under money illusion, market participants fail to take account of the changing value of money.

Fisher's Indirect Channel

- Money illusion rules out the *direct effect* of expected appreciation on interest rates.
- Fisher believed that changes in the value of money could have an *indirect effect* due to the effect of price changes on profits and borrowing.
- Fisher proposed a round-about influence of changing prices on interest rates:

  $\Delta P \rightarrow \Delta$money profits $\rightarrow \Delta$borrowing $\rightarrow \Delta i$

An Implicit CFE

- Using a distributed lag model, Fisher (1930) found that price changes affect nominal interest rates with a long and variable lag.
- Subsequent writers interpreted Fisher's distributed lag model as an *adaptive expectations* model.
- The CFE was born when expected inflation took the place of expected money appreciation in subsequent interpretations of the "Fisher equation." (Q: Who was the first to use the CFE?)

Conclusion

- OFE reflects the theoretical Fisher (rationality of expectations)
- The evolution of the CFE reflects the empirical Fisher (money illusion)
- The two equations are different under uncertainty (and, quite possibly, significantly so).