Exchange rate volatility post-1994: A study of the South African rand

Working Paper (PhD, Ch1)

by

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"Modeling and forecasting the exchange rate remains a hazardous occupation" (*MacDonald & Taylor, 1994)*

Presentation overview

1. Motivation for study
2. Methodology
3. Preliminary tests
4. Estimations & results
5. Parameter instability & structural breaks
6. Concluding remarks (limitations and remedies)

Motivation for study

Currency price volatility & its effects

- affects financial decisions
- impacts on real sector
- policy effects

$\therefore$ **perennial & topical issue**

Methodology

Standard volatility models

- ***Variance and standard deviation***

$$\hat{\sigma}^2 = \frac{1}{n-1}\sum_{t=1}^{n}(r_t - \bar{r})^2, \quad r_t = \ln\left(\frac{e_t}{e_{t-1}}\right), \quad \bar{r} = \frac{1}{n}\sum_{k=1}^{n} r_k$$

- ***High-low variation*** (Parkinson, 1980)

$$\sigma_{HL} = \max(r_t) - \min(r_t)$$

Methodology

(Nonlinear) ARCH Class Models)

$$r_t = \ln\left(\frac{e_t}{e_{t-1}}\right); \qquad \begin{aligned} r_t &= \gamma + \varepsilon_t \\ \varepsilon_t &= h_t z_t \\ z_t &\sim N(0,1) \end{aligned}$$

Methodology

Symmetrical Nonlinear GARCH Models

Basic (restricted) ARCH(p)

$$h_t^2 = \omega + \sum_{k=1}^{p}\alpha_k \varepsilon_{t-k}^2; \qquad {}_u\hat{\sigma}^2 = \frac{\omega}{\left(1 - \sum_{k=1}^{p}\alpha_k\right)}$$

$$\omega > 0; \quad \alpha_k \geq 0 \quad 0 \leq \sum_{k=1}^{p}\alpha_k < 1$$

(Restricted) GARCH(p,q)

$$h_t^2 = \omega + \sum_{k=1}^{p}\alpha_k \varepsilon_{t-k}^2 + \sum_{j=1}^{q}\beta_j h_{t-j}^2 \qquad {}_u\hat{\sigma}^2 = \frac{\omega}{\left(1 - \sum_{k=1}^{p}\alpha_k - \sum_{j=1}^{q}\beta_j\right)}$$

$$p > 0; \quad q \geq 0; \quad \omega > 0; \quad \alpha_k \geq 0; \quad \beta_j \geq 0; \quad \sum_{k=1}^{p}\alpha_k + \sum_{j=1}^{q}\beta_j < 1$$

Slide 1

067 09900669, 2009/09/04

Slide 3

068 Risk & return trade-off: aset valuation, portfolio diversification & optimisation, derivative pricing & risk management
09900669, 2009/09/04

069
Exchange rate instablity can affect fixed investment, economic growth & employment & unemployment.
09900669, 2009/09/04

070 Conspicuous attribute of financial time series is the inherent volatility. Modeling & forecasting time-varying exchange rate volatilit in partcular, reflects its importanc in a host of finacial economics decisions.
09900669, 2009/09/04

071 Exchange rate instability is & should be an important element of exchange rate policy, monetary policy and other economic policy desicions
09900669, 2009/09/04

Slide 4

077 Historically most popular measure of dispersion and volatility
09900669, 2009/09/04

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089
Stationary ARCH
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094 09900669, 2009/09/05

Methodology

IGARCH(p,q)

$$h_t^2 = \sum_{k=1}^{p} \alpha_k \varepsilon_{t-k}^2 + \sum_{j=1}^{q} \beta_j h_{t-j}^2$$

$$p > 0; \quad q \geq 0; \; \alpha_k \geq 0; \; \beta_j \geq 0; \; \sum_{k=1}^{p} \alpha_k + \sum_{j=1}^{q} \beta_j = 1$$

CGARCH(p,q)

$$h_t^2 = m_t + \alpha\left(\varepsilon_{t-1}^2 - m_{t-1}\right) + \beta\left(h_{t-1}^2 - m_{t-1}\right)$$

$$m_t = \omega + \rho\left(m_{t-1} - \omega\right) + \phi\left(\varepsilon_{t-1}^2 - h_{t-1}^2\right)$$

$$h_t^2 = (1-\alpha-\beta)(1-\rho)\omega + (\alpha+\phi)\varepsilon_{t-1}^2 - (\alpha\rho + (\alpha+\beta)\phi)\varepsilon_{t-2}^2 + (\beta-\phi)h_{t-1}^2 - (\beta\rho - (\alpha+\beta)\phi)h_{t-2}^2$$

Methodology

Asymmetrical Nonlinear GARCH Models

EGARCH(p,q,r)

$$\ln h_t^2 = \omega + \sum_{k=1}^{p} \alpha_k \left| \frac{\varepsilon_{t-k}}{h_{t-k}} \right| + \sum_{i=1}^{r} \alpha_i^* \frac{\varepsilon_{t-i}}{h_{t-i}} + \sum_{j=1}^{q} \beta_j \ln h_{t-j}^2$$

TGARCH(p,q,r)

$$h_t^2 = \omega + \sum_{k=1}^{p} \alpha_k \varepsilon_{t-k}^2 + \sum_{i=1}^{r} \alpha_i^* \varepsilon_{t-i}^2 I_{t-i} + \sum_{j=1}^{q} \beta_j h_{t-j}^2$$

Methodology

PGARCH(p,q,r)

$$h_t^{\delta} = \omega + \sum_{k=1}^{p} \alpha_k \left(\left| \varepsilon_{t-k} \right| - \alpha_{t-k}^* \right)^{\delta} + \sum_{j=1}^{q} \beta_j h_{t-j}^{\delta}$$

TCGARCH(p,q,r)

$$h_t^2 = m_t + \alpha\left(\varepsilon_{t-1}^2 - m_{t-1}\right) + \alpha^*\left(\varepsilon_{t-1}^2 - m_{t-1}\right)d_{t-1} + \beta\left(h_t^2 - m_{t-1}\right) + \theta_2 z_{2t}$$

$$m_t = \omega + \rho\left(m_{t-1} - \omega\right) + \phi\left(\varepsilon_{t-1}^2 - h_{t-1}^2\right) + \theta_1 z_{1t}$$

Data Issues

$$r_t = \ln\left(e_t / e_{t-1}\right)$$

- *Frequency:* daily & weekly r_t
- *Sample period:* 13 March 1995 – 30 June 1998
- *Sample length:* 3321 (daily r_t) & 694 (weekly r_t)
- Indirect e_t: NBERs (USD, EUR, GBP & JPY) & NEER
- *Source:* SARB online database

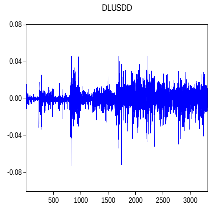
Preliminary Tests

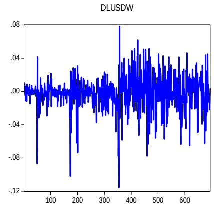
Unit Root tests (r_t)

- *Preliminary tests*: Graph series
- *Formal tests* for (non)stationarity: **unit root tests** (popular)
- Employ 4 different unit root tests, viz.
  - ADF (augmented Dickey-Fuller) test (1979)
  - PP (Phillips-Perron) test (1988)
  - DF-GLS (Dickey-Fuller Generalised Least Squares) proposed by Elliott, Rothenberg, & Stock (ERS) (1996)
  - KPSS (Kwiatkowski, Phillips, Schmidt, & Shin (1992)
- All daily & weekly r_t series are stationary (at 1%, 5% & 10% significance levels); (*1 exception*: weekly $\boldsymbol{r_t^{¥/R}}$ at 1% sign. level*)

Preliminary Tests & Results

Volatility Clustering & ARCH Effects (r_t)





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086 Integrated = Nonstationary
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Nonnormality: Asymmetry & Kurtosis

Table 1: Summary statistics for currency returns (r_t)

Daily returns								
Return	Mean	Med	Max	Min	Std Dev.	Skew	Kurt	JB (Prob)
USD/ZAR	-0.00023	0.00007	0.04637	-0.07263	0.00995	-0.58227	7.74877	3307.12 (0.0000)
EUR/ZAR	-0.00029	0.00024	0.05890	-0.07080	0.01000	-0.49243	7.69592	3184.65 (0.0000)
GBP/ZAR	-0.00030	0.00004	0.05731	-0.06833	0.00985	-0.52104	7.69350	3197.56 (0.0000)
JPY/ZAR	-0.00019	0.00024	0.06239	-0.08535	0.01152	-0.53060	6.69370	2043.11 (0.0000)
NEER	-0.00026	0.00019	0.05516	-0.07228	0.00947	-0.60565	8.65518	4627.02 (0.0000)
Weekly returns								
Return	Mean	Med	Max	Min	Std Dev.	Skew	Kurt	JB (Prob)
USD/ZAR	-0.00114	-0.00025	0.07831	-.011549	0.02064	-0.73673	6.54046	424.63 (0.0000)
EUR/ZAR	-0.00140	0.00013	0.08177	-0.12224	0.02113	-0.94129	6.60589	477.78 (0.0000)
GBP/ZAR	-0.00145	-0.00024	0.08471	-0.12475	0.02041	-0.87219	7.33457	630.38 (0.0000)
JPY/ZAR	-0.00088	0.00122	0.08752	-0.11397	0.02410	-0.70799	5.18136	195.29 (0.0000)
NEER	-0.00124	-0.00007	0.08114	-0.11709	0.01974	-0.98305	7.38214	666.11 (0.0000)

Heteroscedasticity (r_t)

Table 2: ARCH LM Tests for Heteroskedasticity in the Squared Residuals of Currency Returns, ε_t^2

		Daily Returns Residuals					Weekly Returns Residuals				
		LM					LM				
Returns	p	1	5	10	20	36	1	5	10	20	36
USD/ZAR		222.5 (0.000)	368.9 (0.000)	431.0 (0.000)	442.3 (0.000)	457.6 (0.000)	7.7 (0.006)	44.0 (0.000)	48.5 (0.000)	53.3 (0.000)	63.7 (0.003)
EUR/ZAR		193.3 (0.000)	296.9 (0.000)	350.8 (0.000)	358.8 (0.000)	373.2 (0.000)	11.1 (0.001)	54.4 (0.000)	55.5 (0.000)	59.4 (0.000)	67.3 (0.000)
GBP/ZAR		204.8 (0.000)	306.2 (0.000)	381.3 (0.000)	392.8 (0.000)	406.6 (0.000)	15.2 (0.000)	58.1 (0.000)	62.1 (0.000)	67.9 (0.000)	73.4 (0.000)
JPY/ZAR		162.3 (0.000)	248.0 (0.000)	314.4 (0.000)	320.7 (0.000)	334.0 (0.000)	4.9 (0.027)	22.3 (0.001)	30.9 (0.001)	37.7 (0.01)	60.7 (0.006)
NEER		219.1 (0.000)	276.4 (0.000)	396.8 (0.000)	404.3 (0.000)	415.7 (0.000)	13.0 (0.000)	58.6 (0.000)	62.4 (0.000)	66.4 (0.000)	74.0 (0.000)

Note: The LM statistic at lag p is a test statistic for the null hypothesis that there is no ARCH effects up to order $p=36$

Serial Correlation (r_t)

Table 3: Ljung-Box or Modified Q-Statistic Tests for Autocorrelation in Currency Returns

Daily Returns											
		$Q_{LB}(p)r_t^2$					$Q_{LB}(p)\lvert r_t\rvert$				
Returns	p	1	5	10	20	36	1	5	10	20	36
USD/ZAR		221.5 (0.000)	660.0 (0.000)	1058.8 (0.000)	1425.0 (0.000)	1667.0 (0.000)	300.84 (0.000)	1272.5 (0.000)	2339.1 (0.000)	3731.7 (0.000)	5244.6 (0.000)
EUR/ZAR		194.0 (0.000)	493.5 (0.000)	782.6 (0.000)	975.4 (0.000)	1040.8 (0.000)	177.7 (0.000)	592.8 (0.000)	1011.5 (0.000)	1428.5 (0.000)	1656.3 (0.000)
GBP/ZAR		204.9 (0.000)	509.7 (0.000)	877.1 (0.000)	1100.4 (0.000)	1217.3 (0.000)	209.1 (0.000)	727.2 (0.000)	1275.2 (0.000)	1806.9 (0.000)	2236.3 (0.000)
JPY/ZAR		162.0 (0.000)	394.7 (0.000)	650.7 (0.000)	802.1 (0.000)	873.4 (0.000)	141.9 (0.000)	485.2 (0.000)	851.9 (0.000)	1149.8 (0.000)	1329.2 (0.000)
NEER		218.7 (0.000)	574.3 (0.000)	934.0 (0.000)	1182.6 (0.000)	1300.7 (0.000)	241.69 (0.000)	904.6 (0.000)	1607.2 (0.000)	2365.4 (0.000)	2951.0 (0.000)
Weekly Returns											
		$Q_{LB}(p)r_t^2$					$Q_{LB}(p)\lvert r_t\rvert$				
Returns	p	1	5	10	20	36	1	5	10	20	36
USD/ZAR		7.5 (0.006)	58.4 (0.000)	63.3 (0.000)	76.1 (0.000)	98.3 (0.000)	35.5 (0.000)	150.8 (0.000)	207.2 (0.000)	320.5 (0.000)	457.0 (0.000)
EUR/ZAR		10.8 (0.001)	66.9 (0.000)	68.2 (0.000)	78.5 (0.000)	95.4 (0.000)	19.9 (0.000)	64.0 (0.000)	70.7 (0.000)	106.8 (0.000)	147.5 (0.000)
GBP/ZAR		15.1 (0.000	75.9 (0.000)	76.9 (0.000)	85.2 (0.000)	101.7 (0.000)	31.5 (0.000)	90.5 (0.000)	101.1 (0.000)	131.5 (0.000)	181.3 (0.000)
JPY/ZAR		4.8 (0.028)	27.5 (0.000)	33.1 (0.000)	45.5 (0.000)	65.6 (0.000)	13.5 (0.000)	30.4 (0.000)	38.3 (0.000)	52.6 (0.000)	82.0 (0.000
NEER		12.7 (0.000)	76.3 (0.000)	78.3 (0.000)	87.4 (0.000)	106.8 (0.000)	37.3 (0.000)	122.3 (0.000)	148.2 (0.000)	208.5 (0.000)	289.2 (0.000)

Notes: The Q_{LB} statistic at lag p is a test statistic for the null hypothesis that there is no autocorrelation up to order $p = 36$.

Mean Equation Specifications

$$r_t = \gamma_0 + \varepsilon_t$$

$$r_t = \gamma_0 + \gamma_1 r_{t-1} + \varepsilon_t$$

$$r_t = \gamma_0 + \gamma_1 r_{t-1} + \gamma_2 h_t^2 + \varepsilon_t$$

Best Model Selection (MLE)

Table 4: Model in-sample performance (ranking based on lowest BIC value using Student's t distribution)

	Daily Returns Series						Weekly Returns Series					
Model	USD	EUR	GBP	JPY	NEER	Avg. Rank	USD	EUR	GBP	JPY	NEER	Avg. Rank
ARCH(1)	16	16	16	16	16	16	16	16	16	4	16	16
ARCH(2)	15	15	15	15	15	15	15	15	15	3	15	15
ARCH(3)	14	14	14	14	14	14	14	14	14	6	14	14
ARCH(4)	13	13	13	13	13	13	13	13	13	9	13	13
ARCH(5)	12	12	12	12	12	12	12	12	12	11	12	12
ARCH(6)	11	11	11	11	11	11	11	11	11	12	11	11
ARCH(7)	10	10	10	10	10	10	10	10	10	13	10	10
ARCH(8)	9	9	9	9	9	9	9	9	9	16	9	9
ARCH(9)	8	8	8	8	8	8	8	8	8	15	8	8
GARCH(1,1)	**1**	5	4	4	6	5	2	3	2	2	2	2
IGARCH(1,1)	5	7	7	7	7	7	**1**	**1**	3	**1**	**1**	**1**
EGARCH(1,1,1)	**1**	**1**	3	2	2	**1**	4	2	4	14	3	5
TGARCH(1,1,1)	3	4	4	**1**	5	3	3	4	6	5	5	3
PARCH(1,1,1)	4	3	6	3	4	4	5	5	7	8	6	6
CGARCH(1,1)	6*	2	**1**	5*	**1**	2	6	6	**1**	7	4	4
CTARCH (1,1,1)	7*	6	2	6*	3	6	7	7	5	10	7	7

Notes: Models with the same BIC value are given the same numerical ranking; in such instances, the next immediate ranking(s) is (are) omitted.
* Estimated using GED; otherwise Student's t distribution.

Estimation Results

Table 9: Regression Results for Best In-Sample Models Specifications for Daily Returns

	USD G(1,1)	USD EG(1,1)	EUR EG(1,1)	GBP CG(1,1)	JPY TG(1,1)	NEER C(1,1)
	Variance Equation					
α_1	0.147	0.271	0.2139	0.126	0.050	0.126
S.E.	0.014	0.020	0.0238	0.016	0.017	0.015
p-value	0.000	0.000	0.0000	0.000	0.003	0.000
α_1^*	-	-0.029	-0.059	-	0.105	-
S.E.	-	0.013	0.0136	-	0.023	-
p-value	-	0.024	0.0000	-	0.000	-
$\alpha_1 + \alpha_1^*$	-	0.241	0.1545	-	0.155	-
$\alpha_1 - \alpha_1^*$	-	0.301	0.2734	-	-	-
β_1	0.878	0.993	0.9666	0.8169	0.848	0.825
S.E.	0.009	0.003	0.0068	0.0239	0.018	0.022
p-value	0.000	0.000	0.0000	0.0000	0.000	0.000
$\alpha+\beta$:						
ε_{t-1}^+	1.025	1.235	1.1211	0.943	0.898	0.952
ε_{t-1}^-	1.025	1.294	1.2400	0.943	1.004	0.95
ρ_1	-	-	-	1.000	-	1.001
S.E.	-	-	-	0.000	-	0.000
p-value	-	-	-	0.000	-	0.000
ϕ_1	-	-	-	0.006	-	0.010
S.E.	-	-	-	0.003	-	0.004
p-value	-	-	-	0.037	-	0.025
$\rho_1 + \phi_1$	-	-	-	1.006	-	1.011

Estimation Results

Table 10: Risk-return tradeoff (Daily returns)						
	USD G(1,1)	USD EG(1,1)	EUR EG(1,1)	GBP CG(1,1)	JPY TG(1,1)	NEER CG(1,1)
	Mean Equation: Constant, AR(1) and Variance					
γ_0 S.E. *p*-value	-0.000 0.000 0.019	-0.000 0.000 0.008	-0.000 0.000 0.705	-0.000 0.000 0.067	0.000 0.000 0.881	-0.000 0.000 0.348
γ_1 S.E. *p*-value	-0.045 -2.540 0.011	-0.045 0.017 0.009	-0.046 0.017 0.007	-0.040 0.018 0.026	-0.020 0.018 0.262	-0.058 0.018 0.001
γ_2 S.E. *p*-value	3.204 1.531 0.036	2.733 1.550 0.078	1.792 2.576 0.486	5.034 2.463 0.041	1.824 2.526 0.470	4.705 2.025 0.020

Estimation Results

Structural Breaks (returns & parameters)

- *Quandt-Andrews test* (rolling Chow breakpoint test): **1 breakpoint**
- *Recursive Least Squares test* (larger subsets of sample are used): **suggest several structural breaks**
- *CUSUM test* (based on CUSUM of recursive residuals): **suggest parameter instability for prolonged periods implying multiple structural breaks**

Estimation Results

Table 15: Regression Results for Best In-Sample Models Specifications for Daily Returns						
	USD (EGARCH(1,1))		**EUR** (EGARCH(1,1))		**JPY** (TGARCH(1,1))	
	No Breaks	Breaks	No Breaks	Breaks	No Breaks	Breaks
α_1 S.E. *p*-value	0.271 0.020 0.000	0.272 0.022 0.000	0.213 0.023 0.000	0.218 0.027 0.000	0.050 0.017 0.003	0.041 0.017 0.022
α_1^* S.E. *p*-value	-0.029 0.013 0.024	- 0.045 0.014 0.000	-0.059 0.013 0.000	- 0.083 0.015 0.000	0.105 0.023 0.000	0.127 0.026 0.000
$\alpha_1 + \alpha_1^*$	0.241	0.227	0.154	0.135	0.155	0.168
$\alpha_1 - \alpha_1^*$	0.301	0.318	0.273	0.302	-	-
β_1 S.E. *p*-value	0.993 0.002 0.000	0.973 0.005 0.000	0.966 0.006 0.000	0.928 0.013 0.000	0.848 0.018 0.000	0.810 0.024 0.000
$\alpha+\beta$: ε_{t-1}^{+} $\alpha+\beta$: ε_{t-1}^{-}	1.235 1.294	1.201 1.291	1.121 1.240	1.063 1.230	0.898 1.004	0.851 0.979
Structural Break S.E. *p*-value		0.081 0.015 0.000		0.077 0.018 0.000	0.065 0.016 0.000	0.065 0.016 0.000

Concluding Remarks

- Daily r_t exhibit more significant levels of dependence
- Most daily & weekly conditional variance of r_t best modeled by asymmetric GARCH & IGARCH, respectively **when <u>(all)</u> structural breaks are not accounted for**; spuriously high estimates of degree of persistence
- Risk-return tradeoff significant in some returns and slight risk-return tradeoff in others
- **Limitations**
  - Eviews6 MLE implementation concerns
  - asymmetric Student's t distribution
  - accounting for multiple structural breaks
- **Remedial actions**
  - MATLAB
  - Mathematica
- **Forecasting**

Implementing MLE: Practical issues

- Choosing starting values for: $\gamma, \alpha_k, \alpha_k^*, \beta_j$
- Initialising: $\varepsilon_t^2, \sigma_t^2$
- $r_t^I = \bar{r}_t$

Table 8: Variance equation estimation options (numerical accuracy tests)										
Backcasting parameter	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1.0
Maximum iterations	500									
Convergence	1E05	1E-04	1E-03	0.01	0.1	1.0				
Starting coefficient values*	OT	0.8OT	0.5OT	0.3OT	zero					
Optimising algorithm**	M	BHHH								

* OT denotes OLS/TSLS (Ordinary least squares and two-stage least squares respectively)
** M denotes Marquardt and BHHH denotes Berndt-Hall-Hall-Hausmen